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ECON40002
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Advanced Macroeconomics
Tutorial #6: Solutions

Stochastic growth model in DYNARE. Suppose the planner maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}, \qquad 0<\beta<1, \qquad \sigma>0$$

subject to the sequence of resource constraints

$$c_t + k_{t+1} = z_t k_t^{\alpha} + (1-\delta)k_t, \qquad 0<\alpha,\delta<1$$

where c_t, k_t etc denote consumption per worker, capital per worker etc. Productivity follows an AR(2) process in logs

$$\log z_t = \phi_1 \log z_{t-1} + \phi_2 \log z_{t-2} + \varepsilon_t$$

where the innovations ε_t are IID $N(0,\sigma_\varepsilon^2)$.

(a) What are the planner's key optimality conditions for consumption c_t and capital k_{t+1}?

Suppose the following parameter values: $\alpha = 0.3$, $\beta = 0.95$, $\delta = 0.05$, $\sigma = 1$ and $\phi_1 = 1.3$, $\phi_2 = -0.4$, $\sigma_\varepsilon = 0.01$. Use DYNARE to do the following:

(b) Solve for the non-stochastic steady state values of the levels of consumption, capital, output and investment.

(c) Calculate the long-run standard deviations of the log-deviations of consumption, capital, output, investment and productivity. Which of these variables move most closely together? Which of these variables is most volatile? Explain.

(d) Suppose the economy is at steady state and that at $t=0$ there is a 1% innovation to productivity, i.e., $\varepsilon_0 = 0.01$. Calculate and plot the impulse response functions for the log-deviations of consumption, capital, output, investment and productivity for $T=50$ periods after the shock. Explain your findings.

(e) Suppose instead that $\phi_1 = 0.94$ and $\phi_2 = 0$. How if at all do your answers to (b), (c) and (d) change? Explain.

SOLUTIONS

(a) Using $u'(c) = c^{-\sigma}$, the planner's key optimality conditions are, as usual, the consumption Euler equation

$$c_t^{-\sigma} = \beta \mathbb{E}_t \left\{ c_{t+1}^{-\sigma} \left[z_{t+1} \alpha k_{t+1}^{\alpha-1} + 1 - \delta \right] \right\}$$

and the resource constraint

$$c_t + k_{t+1} = z_t k_t^{\alpha} + (1-\delta) k_t$$

The attached Dynare file `tutorial5.mod` solves the model with the given parameters.

(b) This gives the steady state values

```
STEADY-STATE
c                  0.301697
k                  1.53234
y                  0.459702
i                  -1.46339
z                  0
```

These are in logs, so for example $\log \bar{c} = 0.301697$ means $\bar{c} = \exp(0.301697) = 1.3522$. We can do this for the whole vector of steady state values by the command `exp(oo_.steady_state)`

```
STEADY-STATE (in levels)
   c                  1.3522
   k                  4.6290
   y                  1.5836
   i                  0.2314
   z                  1.0000
```

(c) We get the standard deviations

VARIABLE	MEAN	STD. DEV.	VARIANCE
c	0.3017	0.0302	0.0009
k	1.5323	0.0399	0.0016
y	0.4597	0.0377	0.0014
i	-1.4634	0.1101	0.0121
z	0.0000	0.0294	0.0009

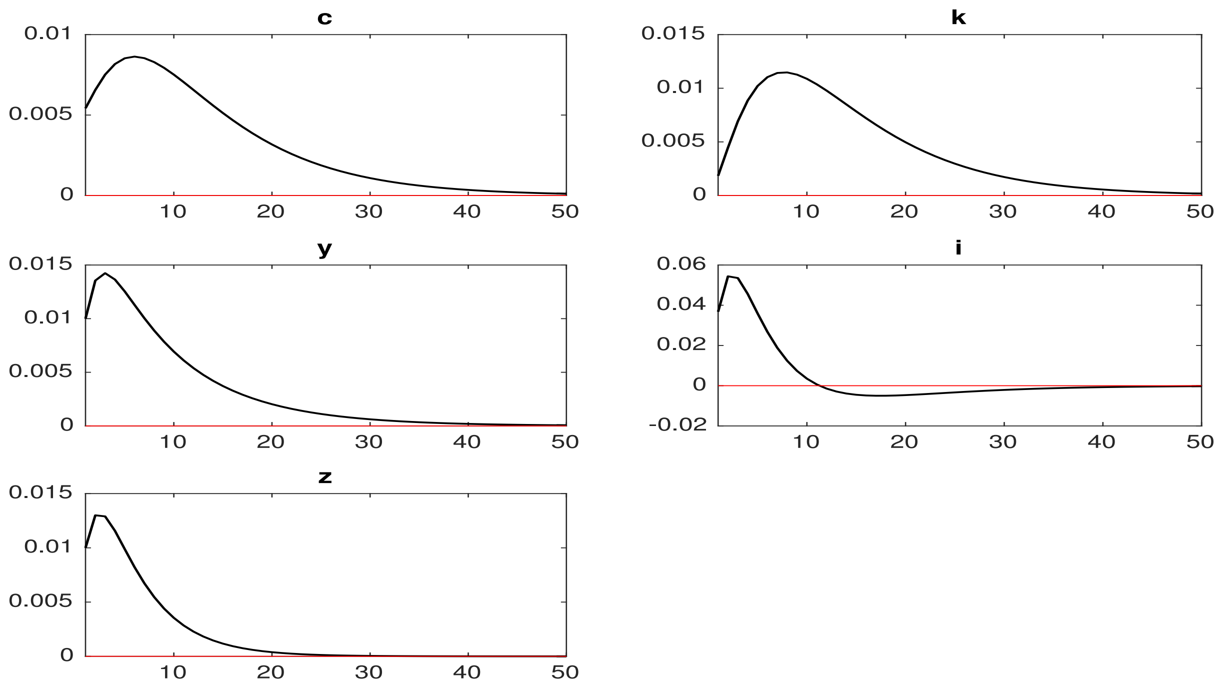
and the correlation matrix

Variables	c	k	y	i	z
c	1.0000	0.9765	0.9392	0.5957	0.8192
k	0.9765	1.0000	0.8491	0.4225	0.6834
y	0.9392	0.8491	1.0000	0.8353	0.9659
i	0.5957	0.4225	0.8353	1.0000	0.9478
z	0.8192	0.6834	0.9659	0.9478	1.0000

Capital and consumption are the most closely correlated, followed by output and productivity, then investment and productivity, then consumption and output. Investment is by far the most volatile, then capital, then output, then consumption. Given the planner's consumption-smoothing motive, it's intuitive that consumption is smoother than output.

(d) The Dynare output gives us the impulse response functions for a 1 standard deviation innovation in productivity. The attached figure shows the results. Note this is the same as a $\varepsilon_0 = 0.01$ shock because $\sigma_\varepsilon = 0.01$, otherwise we would have to rescale things.

Note the hump-shaped impulse response of productivity — it has a first-order autocorrelation $\phi_1 > 1$ with an offsetting second-order autocorrelation $\phi_2 < 0$ that drags it back. This also leads to a hump-shaped response of output.



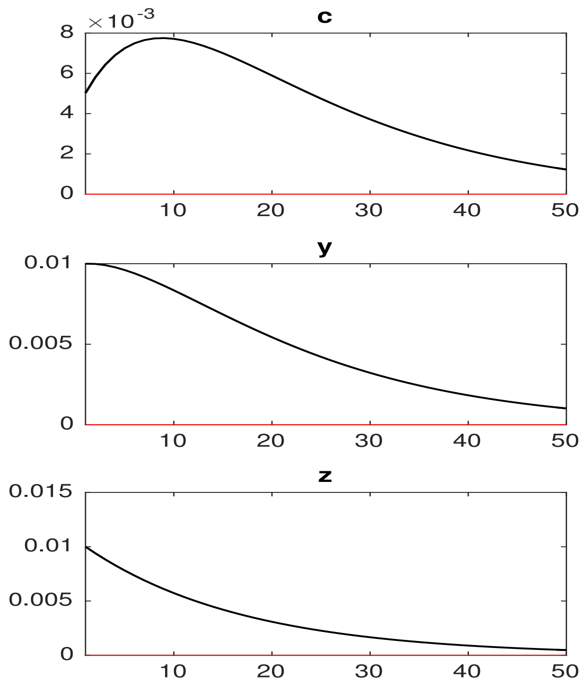
(e) We are now back to our usual AR(1) process for productivity. The steady state is unchanged but we get different fluctuations around steady state

VARIABLE	MEAN	STD. DEV.	VARIANCE
c	0.3017	0.0362	0.0013
k	1.5323	0.0423	0.0018
y	0.4597	0.0400	0.0016
i	-1.4634	0.0815	0.0066
z	0.0000	0.0293	0.0009

with the correlation matrix

Variables	c	k	y	i	z
c	1.0000	0.9852	0.9764	0.6883	0.9145
k	0.9852	1.0000	0.9250	0.5539	0.8316
y	0.9764	0.9250	1.0000	0.8287	0.9803
i	0.6883	0.5539	0.8287	1.0000	0.9230
z	0.9145	0.8316	0.9803	0.9230	1.0000

Note that the long-run volatility of productivity that we feed in is unchanged (this was by the 'right' choice of $\phi_1 = 0.94$). So any changes come not from the long-run volatility but from the 'temporal composition' of that volatility. In particular, we now find that consumption is more correlated than with the AR(2). Investment is less volatile than with the AR(2) while consumption is more volatile. The AR(2) imparts some more predictable low-frequency volatility that the planner can smooth out. Switching the process to an equally volatile AR(1) prevents the planner from taking advantage of this greater predictability.



c
×10-3
8
6
4
2
0
k
0.01
0.005
0
y
0.01
0.005
0
i
0.04
0.02
0
-0.02
z
0.015
0.01
0.005
0
10
20
30
40
50