

Advanced Macroeconomics Tutorial #6

Stochastic growth model in DYNARE. Suppose the planner maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}, \quad 0 < \beta < 1, \quad \sigma > 0$$

subject to the sequence of resource constraints

$$c_t + k_{t+1} = z_t k_t^\alpha + (1 - \delta)k_t, \quad 0 < \alpha, \delta < 1$$

where c_t, k_t etc denote consumption per worker, capital per worker etc. Productivity follows an AR(2) process in logs

$$\log z_t = \phi_1 \log z_{t-1} + \phi_2 \log z_{t-2} + \varepsilon_t$$

where the innovations ε_t are IID $N(0, \sigma_\varepsilon^2)$.

- (a) What are the planner's key optimality conditions for consumption c_t and capital k_{t+1} ?

Suppose the following parameter values: $\alpha = 0.3$, $\beta = 0.95$, $\delta = 0.05$, $\sigma = 1$ and $\phi_1 = 1.3$, $\phi_2 = -0.4$, $\sigma_\varepsilon = 0.01$. Use DYNARE to do the following:

- (b) Solve for the non-stochastic steady state values of the levels of consumption, capital, output and investment.
- (c) Calculate the long-run standard deviations of the log-deviations of consumption, capital, output, investment and productivity. Which of these variables move most closely together? Which of these variables is most volatile? Explain.
- (d) Suppose the economy is at steady state and that at $t = 0$ there is a 1% innovation to productivity, i.e., $\varepsilon_0 = 0.01$. Calculate and plot the impulse response functions for the log-deviations of consumption, capital, output, investment and productivity for $T = 50$ periods after the shock. Explain your findings.
- (e) Suppose instead that $\phi_1 = 0.94$ and $\phi_2 = 0$. How if at all do your answers to (b), (c) and (d) change? Explain.