ECON40002
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Advanced Macroeconomics
Tutorial #5: Solutions

Monopolistic competition, markups, and profits. Suppose a final good Y is produced by *perfectly competitive* firms using a CES bundle of intermediate goods

$$Y = \left(\int_0^N y(i)^{\frac{\theta-1}{\theta}}\, di\right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1$$

The final good firms buy intermediate goods at prices $p(i)$ from intermediate producers $i \in [0, N]$. The intermediate producers are *monopolistically competitive* and choose prices $p(i)$ and output $y(i)$ to maximize profits understanding their market power.

Suppose intermediate producers need only labor to produce and have production function

$$y(i) = Al(i), \qquad A > 0$$

and take the wage rate W as given. There is a fixed (inelastic) supply of labor $L > 0$.

(a) Let $C(y)$ denote the cost function for each intermediate producer. Show that $C(y)$ is linear

$$C(y) = cy, \qquad c > 0$$

and derive a formula for the marginal cost c in terms of the wage W and other parameters.

(b) Show that the optimal price chosen by each producer is given by

$$p = \mu c$$

where $\mu > 1$ is the (gross) markup over marginal cost. Solve for the markup μ.

Now consider a symmetric equilibrium with $y(i) = y$, $p(i) = p$ etc for all $i \in [0, N]$.

(c) Solve for a symmetric equilibrium. In particular, solve for the equilibrium wage W and equilibrium output Y.

(d) Let $s_L = WL/Y$ denote the labor share and s_Π denote the profit share. Solve for s_L and s_Π. Do higher markups increase the profit share? Explain.

Now suppose that to operate each intermediate producer must pay a *fixed cost* $f > 0$ in units of labor. That is, to produce y units of output the producer needs $\ell(y) \equiv f + y/A$ units of labor. Suppose also that there is an unlimited number of potential entrants and that firms enter if they are willing to pay this fixed cost f.

(e) Solve for the equilibrium number of producers N in terms of the fixed cost f and other parameters. Give intuition for your result.

Solutions:

(a) The cost function for each producer is defined by

$$C(y) \equiv \min_{l} \Big[Wl \mid Al = y \Big]$$

Which is clearly solved by choosing labor $l = \ell(y) = y/A$ so that

$$C(y) = W\ell(y) = \frac{W}{A}\, y$$

Hence the constant marginal cost is $c = W/A$.

(b) To begin with, taking prices $p(i)$ as given, final good producers choose the bundle $y(i)$ to maximize

$$Y - \int_0^N p(i)y(i)\, di$$

subject to

$$Y = \left(\int_0^N y(i)^{\frac{\theta-1}{\theta}}\, di \right)^{\frac{\theta}{\theta-1}}$$

In other words they choose $y(i)$ to maximize

$$\left(\int_0^N y(i)^{\frac{\theta-1}{\theta}}\, di \right)^{\frac{\theta}{\theta-1}} - \int_0^N p(i)y(i)\, di$$

For each $i \in [0, N]$ the first order condition can be written

$$\frac{\theta}{\theta-1} \left(\int_0^N y(i)^{\frac{\theta-1}{\theta}}\, di \right)^{\frac{\theta}{\theta-1}-1} \frac{\theta-1}{\theta} y(i)^{\frac{\theta-1}{\theta}-1} - p(i) = 0$$

Simplifying this gives

$$\left(\int_0^N y(i)^{\frac{\theta-1}{\theta}}\, di \right)^{\frac{1}{\theta-1}} y(i)^{-\frac{1}{\theta}} = p(i)$$

But by the definition of Y this is the same as

$$Y^{\frac{1}{\theta}} y(i)^{-\frac{1}{\theta}} = p(i)$$

This implies that the demand curve facing each intermediate producer is

$$y(i) = p(i)^{-\theta}\, Y$$

Each intermediate producer internalizes their market power and chooses $p(i)$ to maximize profits

$$\pi(i) \equiv \max_{p(i)} \Big[p(i)y(i) - cy(i) \; \Big| y(i) = p(i)^{-\theta} Y \Big]$$

which implies

$$\pi(i) = \max_{p(i)} \Big[p(i)^{1-\theta} - cp(i)^{-\theta} \Big] Y$$

This profit maximization problem has the first order condition

$$\Big[(1-\theta)p(i)^{-\theta} + \theta c p(i)^{-\theta-1}\Big] Y = 0$$

which simplifies to

$$(1-\theta) + \theta c p(i)^{-1} = 0$$

or

$$p(i) = \frac{\theta}{\theta - 1}\, c$$

Hence indeed each symmetric producer charges a price $p = \mu c$ that is a markup $\mu > 1$ over marginal cost $c > 0$ with the gross markup

$$\mu = \frac{\theta}{\theta - 1} > 1$$

(c) In a symmetric equilibrium with $y(i) = y$, $p(i) = p$ etc we have from the definition of aggregate output Y that

$$Y = N^{\frac{\theta}{\theta-1}} y$$

But also from the zero profit condition for final goods producers we have

$$Y = Npy$$

So together these imply

$$Np = N^{\frac{\theta}{\theta-1}}$$

or

$$p = N^{\frac{1}{\theta-1}}$$

Since here N is exogenous, this tells us what the *equilibrium* price of each intermediate must be. The calculation in part (b) above is conceptually different in that it tells us that $p = \mu c$ is *optimal* for each intermediate producer, but since $c = W/A$ is endogenous (due to its dependence on the endogenous wage W) this optimality condition doesn't tell us what prices are in equilibrium. But now we can combine these conditions to get

$$p = N^{\frac{1}{\theta-1}} = \mu c = \frac{\theta}{\theta - 1}\frac{W}{A}$$

which tells us that the equilibrium wage W must be

$$W = \frac{\theta - 1}{\theta} N^{\frac{1}{\theta-1}} A$$

Now observe from the labor market clearing condition

$$L = Nl = N\frac{y}{A}$$

So in equilibrium each intermediate produces output

$$y = \frac{AL}{N}$$

Equilibrium aggregate output Y is given by

$$Y = Npy = N\, N^{\frac{1}{\theta-1}} \frac{AL}{N} = N^{\frac{1}{\theta-1}} AL$$

(d) The equilibrium labor share is given by

$$s_L \equiv \frac{WL}{Y} = \frac{W}{Y/L} = \frac{\frac{\theta-1}{\theta} N^{\frac{1}{\theta-1}} A}{N^{\frac{1}{\theta-1}} A} = \frac{\theta-1}{\theta} = \frac{1}{\mu}$$

Higher markups $\mu = \frac{\theta}{\theta-1}$ therefore reduce the labor share.

The profits of each intermediate producer are

$$\pi = (p-c)y = (\mu-1)cy = (\mu-1)\frac{W}{A}y = (\mu-1)\frac{W}{A}\frac{AL}{N} = (\mu-1)\frac{WL}{N}$$

So aggregate profits are

$$\Pi = N\pi = (\mu-1)WL$$

So the profit share is

$$s_\Pi \equiv \frac{\Pi}{Y} = (\mu-1)\frac{WL}{Y} = (\mu-1)\frac{1}{\mu} = 1-\frac{1}{\mu} = 1-s_L$$

So as we would expect, given that there is no physical capital, $s_L + s_\Pi = 1$. Since higher markups reduce the labor share s_L, higher markups increase the profit share $s_\Pi = 1 - s_L$.

(e) With the fixed cost f in units of labor, the profits of each producer become

$$\pi = (p-c)y - Wf = (\mu-1)\frac{W}{A}y - Wf$$

Now free entry drives profits to zero (total revenue equals total cost), so

$$(\mu-1)\frac{W}{A}y = Wf$$

which pins down how much each intermediate produces

$$y = \frac{Af}{\mu-1}$$

To pin down the number of producers N we turn to the labor market clearing condition

$$L = Nl$$

Since the fixed cost is in units of labor, the labor used by each producer is $l = \ell(y) = f + y/A$ and given our solution for y above

$$l = f + \frac{y}{A} = f + \frac{f}{\mu-1} = \left(1 + \frac{1}{\mu-1}\right) f = \frac{\mu}{\mu-1} f = \theta f$$

So the number of producers is

$$N = \frac{L}{l} = \frac{L}{\theta f}$$

This is Adam Smith's solution — '*the division of labor is limited by the extent of the market*'. The number of producers is the product of two terms L/f and $1/\theta$. The first term, L/f, is simply how many firms can feasibly be supported given that each firm requires f labor and there is total labor supply L. The number of firms is less than this, but how much less depends on the second term, $1/\theta$. The second term is a measure of how differentiated the producers are (i.e., θ is a measure of how substitutable they are). When $1/\theta$ is relatively high, near 1, the producers are relatively highly differentiated and the economy can sustain a greater number of producers, close to the maximum feasible number L/f.

In this equilibrium, profits per producer π and hence aggregate profits Π are zero. The labor share is $s_L = 1$ regardless of the markup μ. The excess of total revenue over total *variable* costs, $py - cy = (\mu - 1)cy = (\mu - 1)Wy/A > 0$ simply covers the payments to the fixed factor Wf.