Faculty of Business and Economics
Department of Economics

ECON40002
Chris Edmond

Advanced Macroeconomics

Tutorial #4

Linear growth. Consider a continuous time Ramsey-Cass-Koopmans model. The planner seeks to maximize the intertemporal utility function

$$\int_0^\infty e^{-\rho t} \log(c(t))\, dt, \qquad \rho > 0$$

Output per person is given by the linear production function $y = Ak$ so that the planner's flow resource constraint is

$$\dot{k}(t) = (A - \delta)k(t) - c(t)$$

given initial $k(0) > 0$. Assume that the marginal product of capital is greater than the depreciation rate, $A > \delta$.

(a) Setup a Hamiltonian for this problem. Derive the key optimality conditions for $c(t)$ and $k(t)$.

(b) Let $r \equiv A - \delta$ and suppose $r > \rho$. Use a phase diagram to explain the dynamics of $c(t)$ and $k(t)$. Do these dynamics generally converge to a steady state c^* and k^*? Why or why not? Explain. How if at all would your answer change if instead $r < \rho$?

(c) Solve for the optimal $c(t)$ and $k(t)$ starting from initial conditions $c(0)$ and $k(0)$. What value must $c(0)$ have if these trajectories are to be optimal? Show that your solution satisfies the transversality condition.

(d) Show that the dynamics of $c(t)$ and $k(t)$ can be written as a linear system of the form

$$\begin{pmatrix} \dot{c}(t) \\ \dot{k}(t) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} c(t) \\ k(t) \end{pmatrix}$$

Express the coefficients a_{11} etc in terms of model parameters. Do the eigenvalues of this coefficient matrix imply stable or unstable dynamics? Explain.