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Advanced Macroeconomics
Tutorial #3

Government consumption — a simple case. Suppose the planner seeks to maximize the intertemporal utility function

$$\sum_{t=0}^{\infty} \beta^t \, u(c_t, g_t), \qquad 0 < \beta < 1$$

subject to the sequence of resource constraints

$$c_t + g_t + k_{t+1} = F(k_t, A) + (1 - \delta)k_t, \qquad 0 < \delta < 1$$

given initial $k_0 > 0$. Here g_t denotes government purchases that provide utility (think of this as public services). The period utility function $u(c, g)$ has positive but diminishing marginal utility for each good. All variables are in per worker units.

(a) Derive optimality conditions that characterize the solution to the planner's problem. Give intuition for those optimality conditions. Explain how these optimality conditions pin down the dynamics of c_t, g_t and k_t.

(b) Derive expressions characterizing steady state c^*, g^*, k^*, y^* in this economy. Do these steady state values depend on the period utility function? Explain.

Now suppose that the production function is Cobb-Douglas, $y = F(k, A) = k^\alpha A^{1-\alpha}$ with $0 < \alpha < 1$ and that the utility function is $u(c, g) = (1 - \gamma) \log(c) + \gamma \log(g)$ with $0 < \gamma < 1$.

(c) Solve for steady state values c^*, g^*, k^*, y^* and for the shares c^*/y^* and g^*/y^* in terms of the parameters. How do these depend on γ? How do these depend on A? Explain. Suppose the specific values: $\alpha = 0.3$, $\beta = 0.99$, $\gamma = 0.3$, $\delta = 0.02$ and $A = 1$. Calculate c^*, g^*, k^*, y^*.

(d) Log-linearize the optimality conditions from (a) around the steady-state. Guess that in log-deviations

$$\hat{c}_t = \psi_{ck}\hat{k}_t$$
$$\hat{g}_t = \psi_{gk}\hat{k}_t$$

and

$$\hat{k}_{t+1} = \psi_{kk}\hat{k}_t$$

Use the method of undetermined coefficients and the parameter values from (c) to calculate $\psi_{ck}, \psi_{gk}, \psi_{kk}$. How if at all do these differ from the answers you would get if there was no government consumption, $\gamma = 0$? Explain.

(e) Suppose the economy is at steady state then suddenly at $t = 0$ there is a 1% permanent increase in the level of productivity from $A = 1$ to $A' = 1.01$. Explain *qualitatively* the transitional dynamics of the economy as it adjusts to its new long run values. What happens to the ratio of private to public consumption c_t/g_t?