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ECON40002
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Advanced Macroeconomics
Tutorial #2

1. **Ramsey-Cass-Koopmans model.** Suppose the planner seeks to maximize the intertemporal utility function

$$\sum_{t=0}^{\infty} \beta^t \, u\Big(\frac{C_t}{L}\Big)L, \qquad 0<\beta<1$$

subject to the sequence of resource constraints

$$C_t + K_{t+1} = F(K_t, AL) + (1-\delta)K_t, \qquad 0<\delta<1$$

given initial $K_0 > 0$. Suppose for simplicity that the labor force L and the level of productivity A are constant. Let c_t, k_t, y_t etc denote consumption, capital, output etc in *per worker* units. Suppose that the period utility function and the production function have their usual properties.

   (a) Derive optimality conditions that characterize the solution to the planner's problem. Give intuition for those optimality conditions. Explain how these optimality conditions pin down the dynamics of c_t and k_t.

   (b) Now suppose that the production function is Cobb-Douglas, $Y = F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$ with $0 < \alpha < 1$. Derive expressions for the steady state values c^*, k^* and y^* in this economy. How do these steady state values depend on the period utility function? Explain.

   (c) What is the steady state savings rate in this economy? Explain how the steady state savings rate depends on the parameters α, β, δ and the level of productivity A. Give intuition for your answers.

   (d) Suppose the economy is initially in the steady state you found in (b). Suppose the economy becomes more patient with the discount factor increasing from β to $\beta' > \beta$. Use a phase diagram to explain (i) how this change affects the long-run values of consumption, capital and output, and (ii) how the economy *transitions* to these new long-run values. How would your answers differ if the discount factor fell from β to $\beta' < \beta$?

2. **Isoelastic utility.** Consider the utility function

$$u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}, \qquad \sigma > 0$$

   (a) Show that $u(c)$ is strictly increasing and strictly concave.

   (b) Show that the relative curvature of the utility function is

$$-\frac{u''(c)c}{u'(c)} = \sigma$$

   independent of the level of consumption.

   (c) Consider the case $\sigma \to 1$. Show that this corresponds to $u(c) \to \log c$. [*Hint*: what is the antiderivative of c^{-1}?]