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Advanced Macroeconomics
Tutorial #11

1. **Optimal insurance and deposit contracts.** Consider the Diamond-Dybvig model. There are three dates $\{0, 1, 2\}$ and a unit mass of ex ante identical investors and a single bank. Each of the investors has an endowment of 1 to invest at date $t = 0$. The type of each investor is revealed at date $t = 1$. A fraction $\alpha \in (0, 1)$ are *impatient* and consume only at $t = 1$. The remaining fraction are *patient* and indifferent between consuming at either $t = 1$ or $t = 2$. An individual's realised type is her own private information.

   Funds invested for *two* periods earn a gross return $R > 1$ (an *illiquid project*). Funds invested for only one period earn a gross return of 1 (i.e., the investor just gets their funds back).

   Each investor has the CRRA utility function

   $$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \qquad \sigma > 1$$

   (a) Set up the optimization problem the solution of which gives the efficient amount of risk-sharing (optimal insurance) between impatient and patient investors.

   (b) Suppose the following parameter values: $\alpha = 0.5$, $R = 4$ and $\sigma = 2$. Using these parameter values, solve the optimization problem for the payments c_1^*, c_2^* to impatient and patient investors.

   (c) Explain how the optimal insurance scheme can be implemented by a liquid deposit contract with the bank that pays returns r_1, r_2 on dates $t = 1$ and $t = 2$ respectively. What would the values of r_1, r_2 have to be?

   (d) Calculate the ex ante expected utility to an investor who enters into this deposit contract. Is this higher or lower than the ex ante expected utility of an investor who just invests and holds the illiquid asset? Explain. How would your answer change (if at all) if the investors were risk neutral (e.g., $u(c) = c$)? Explain.

   (e) Explain the *sequential service constraint* facing the bank if it offers deposit contracts. Explain why the bank is prone to a run. If the return on the deposit contract paid in the first period r_1 is the value calculated in part (c), what is the maximum number of withdrawals f^* beyond which any individual patient investor will find it optimal to withdraw? [the '*tipping point*']

2. **Structured finance.** Suppose there are two bonds and that each pays \$1 cash or not. The probability of getting \$1 is 0.8 and is independent across bonds.

   (a) Explain how a financial intermediary can sell prioritized junior j and senior s claims to \$1 against the possible cash flows from a portfolio of these two bonds. In your answer, give the possible realizations of the cash flows, the probabilities of these events, and the payments made to junior and senior claims in each event. How much would a risk neutral investor be prepared to pay for the j and s claims. Is this more or less than they would pay for the underlying bonds? Explain.

(b) Now suppose there are *two pools each of two bonds* each as in part (a) above. Each pool has junior and senior claims. Explain how a financial intermediary can sell prioritized junior j_j and senior s_j claims to $1 against the possible cash flows from a portfolio formed from the junior tranches j_1 and j_2 from each pool. What pattern of cash flows leads to senior claim in the second round of securitization being paid or not paid? Give the possible realizations of the cash flows, the probabilities of these events, and the payments made to the j_j and s_j claims from the second round of securitization. Would a risk neutral investor pay more for a senior claim in the first round of securitization (s_1 or s_2) or for a senior claim in the second round (s_j)? Explain.

(c) Now suppose there are two bonds as in part (a) except that the underlying bonds payments are perfectly positively correlated. Give the possible realizations of the cash flows, the probabilities of these events, and the payments made to junior and senior claims in each event. Would a risk neutral investor be prepared to pay a premium for senior claims? Explain. What if the underlying bond payments are instead perfectly *negatively* correlated, would your answers change? Would a risk averse investor view things differently?