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Advanced Macroeconomics
Tutorial #10

1. **Practice with Bellman values.** Consider a discrete time setting $t = 0, 1, 2, \ldots$ where a risk-neutral worker receives period utility w_t from a stream of wage payments. Let W_t denote the present value of these payments discounted at constant rate $r > 0$

$$W_t \equiv \sum_{s=t}^{\infty} e^{-r(s-t)}\, w_s$$

(supposing the sum is well-defined, e.g., that $w_t \to \bar{w} > 0$ as $t \to \infty$).

(a) Show that the present value W_t satisfies the difference equation

$$W_t = w_t + e^{-r}\, W_{t+1} \tag{1}$$

(b) Now let the period length be $\Delta > 0$ so that $t = 0, \Delta, 2\Delta, \ldots$ with total wage payment $w_t\Delta$ over a period of length Δ. Explain why the present value W_t now satisfies the difference equation

$$W_t = w_t\Delta + e^{-r\Delta}\, W_{t+\Delta}$$

(c) Now let the period length $\Delta \to 0$. Show that in this continuous time limit, the present value satisfies the differential equation

$$rW(t) = w(t) + \dot{W}(t) \tag{2}$$

Now consider again the simple discrete time $t = 0, 1, 2, \ldots$ setting with wage payments w_t. Suppose that the wage is stochastic and can take on two values, $w_t \in \{w_l, w_h\}$ with $0 < w_l < w_h$. Suppose that if the current wage is w_l then with probability p_{ll} the wage next period remains w_l but with probability $1 - p_{ll}$ the wage next period jumps up to w_h. Likewise if the current wage is w_h then with probability p_{hh} the wage next period remains w_h but with with probability $1 - p_{hh}$ the wage next period drops to w_l. In short

$$p_{ll} = \text{Prob}[w_{t+1} = w_l \,|\, w_t = w_l]$$
$$p_{hh} = \text{Prob}[w_{t+1} = w_h \,|\, w_t = w_h]$$

(d) Let W_t^l denote the present value of wages conditional on $w_t = w_l$. Likewise let W_t^h denote the present value of wages conditional on $w_t = w_h$. What system of difference equations do these values solve?

(e) Now again consider periods of length $\Delta > 0$ and suppose that over a period of length Δ the probability of remaining in state l is $p_{ll} = e^{-\lambda_h\Delta}$ for some $\lambda_h > 0$ and similarly the probability of remaining in state h is $p_{hh} = e^{-\lambda_l\Delta}$ for $\lambda_l > 0$. Again let the period length $\Delta \to 0$. What system of differential equations do $W^l(t), W^h(t)$ solve? How do the present values depend on λ_l, λ_h in steady state? Explain.

2. **Mortensen-Pissarides model.** Consider a search model of the labor market in continuous time $t \geq 0$. Risk neutral workers and firms discount at constant rate $r > 0$. Workers and firms are matched via a standard constant-returns-to-scale matching function $F(u, v)$ where $u(t)$ denotes the unemployment rate and $v(t)$ the vacancy rate at time t. When a match forms, a firm is able to produce a constant amount of output $z > 0$. The worker receives a wage of $w(t)$ and the firm makes a flow profit of $z - w(t)$. Job matches between workers and firms are destroyed at an exogenous rate $\delta > 0$. Firms can create jobs by posting vacancies with a flow cost κz proportional to z. There is free-entry into job creation. When unemployed, workers receive constant flow utility $b \leq w(t)$ from unemployment benefits.

   (a) Let $V(t), J(t)$ denote the value to a firm of a vacancy and a filled job respectively and let $U(t), W(t)$ denote the value to a worker of unemployment and employment respectively. What are the Bellman equations that describe these four values? Provide an intuitive explanation for each of these equations.

   (b) Explain how the Bellman equations for $U(t), W(t)$ relate to those you derived in Question 1 part (e). Give as much detail as you can.

   Now suppose that wages are determined by Nash-Bargaining between a worker and firm such that in equilibrium the worker's surplus is a constant fraction $\beta \in (0, 1)$ of the total match surplus

$$W(t) - U(t) = \beta S(t), \qquad S(t) \equiv W(t) - U(t) + J(t) - V(t)$$

   Suppose also that free entry drives the value of a vacancy to $V(t) = 0$ for all t.

   (c) Let the matching function be $F(u, v) = u^\alpha v^{1-\alpha}$. Explain how the steady state wage, labor market tightness $\theta = v/u$ and unemployment rate are determined.

   (d) Now suppose the parameter values $r = 0.01$, $z = \kappa = 1$, $b = 0.4$ and $\alpha = \beta = 0.5$. Calculate the steady state wage, labor market tightness, unemployment rate, vacancy rate, and vacancy filling rate $q = F(u, v)/v$. If productivity increases from $z = 1$ to $z = 1.1$ what happens to each of these variables? What about if r increases from $r = 0.01$ to $r = 0.02$? Give intuition for your answers.

   (e) Now consider what happens if the wage is fixed at some exogenous level $\bar{w}$. Suppose in particular that $\bar{w}$ equals the value you found in part (d) for $z = 1$ and then productivity increases to $z = 1.1$ holding the wage fixed at $\bar{w}$. What happens to labor market tightness and the unemployment and vacancy rates? How if at all do your answers differ to those you found in part (d)? Explain.