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Advanced Macroeconomics
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Advanced Macroeconomics: Problem Set #3

Due Wednesday May 29 in class

1. **Inflation dynamics under optimal monetary policy.** Suppose the monetary authority cannot commit to future actions and seeks to minimize the one-period loss function

$$L = \frac{1}{2}\left(\hat{x}_t^2 + \lambda \hat{\pi}_t^2\right), \qquad \lambda > 0$$

subject to the new Keynesian Phillips curve

$$\hat{\pi}_t = \beta \mathbb{E}_t \left\{\hat{\pi}_{t+1}\right\} + \kappa \hat{x}_t + u_t$$

where the cost push shocks u_t follow a stationary AR(1) process

$$u_{t+1} = \phi u_t + \epsilon_{t+1}, \qquad 0 \leq \phi < 1$$

and where the innovations ϵ_t are IID normal with mean zero and variance σ_u^2.

(a) Derive the monetary authority's optimal discretionary policy for inflation and the output gap and the dynamics of inflation implied by this discretionary policy.

(b) Guess that, in this scenario, inflation and the output gap are linear in the cost push shock, $\hat{\pi}_t = \psi_{\pi u} u_t$ and $\hat{x}_t = \psi_{xu} u_t$ for two coefficients $\psi_{\pi u}$ and ψ_{xu}. Use the method of undetermined coefficients to solve for $\psi_{\pi u}$ and ψ_{xu}. Explain intuitively how inflation and the output gap respond to a cost push shock.

Now suppose the parameter values are $\kappa = 0.15$, $\beta = 0.95$, $\phi = 0.8$, $\sigma_u = 0.015$ and that the monetary authority's weight on inflation is $\lambda = 1$.

(c) Simulate a sequence of cost push shocks u_t of length $T = 250$ and use this to generate simulated sequences of inflation and the output gap. Does a scatterplot of inflation and the output gap reveal any relationship between inflation and the output gap? Are times of economic slack times of low inflation? Why or why not? Give intuition for your results

(d) Suppose you estimate a reduced form Phillip Curve relationship by OLS

$$\hat{\pi}_t = a + b\hat{x}_t + \varepsilon_t$$

Is the estimated relationship between inflation and the output gap upward or downward sloping? Why? How do the OLS coefficients a, b relate to the underlying parameters of the model? How do the OLS residuals ε_t relate to the underlying shocks? Explain.

2. **Discretionary monetary policy in a liquidity trap.** Consider a continuous-time new Keynesian model with dynamic IS curve

$$\dot{x}(t) = \frac{1}{\sigma}(i(t) - \pi(t) - r^n(t))$$

and Phillips curve

$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

Here $x(t)$ denotes the log output gap, $\pi(t)$ the instantaneous inflation rate, $i(t)$ the nominal interest rate, and $r^n(t)$ the natural real rate. The parameters σ, ρ, κ have their usual meanings.

Suppose that the natural real rate follows

$$r^n(t) = \begin{cases} \underline{r} & t \in [0, T) \\ \\ \overline{r} & t \in [T, \infty) \end{cases} \qquad \text{with} \quad \underline{r} < 0 < \overline{r}$$

for some given horizon $T > 0$ with $\underline{r}$ sufficiently negative that the ZLB constraint on $i(t)$ is binding for $t \in [0, T)$. Suppose that for $r^n(t) = \overline{r} > 0$ it is possible to implement $i(t) = \overline{r}$ with a sufficiently reactive interest rate rule.

In this question you will study outcomes under monetary policy *discretion*.

   (a) Explain, qualitatively, how inflation $\pi(t)$ and the output gap $x(t)$ are determined both during and after the liquidity trap under monetary policy discretion.

Now suppose the parameter values: $\sigma = 1$, $\kappa = 0.5$, and $\rho = 0.05$ per year with $\overline{r} = +\rho$, $\underline{r} = -\rho$ and $T = 2$ years.

   (b) Solve for the time-path of $\pi(t)$ and $x(t)$ during and after the liquidity trap. Plot $\pi(t)$ and $x(t)$ against time and against each other. Explain your findings.

   (c) How would your answers differ if instead $\kappa = 0.25$? or $\kappa = 0.75$? What if instead $T = 1$ years? or $T = 3$ years? And what about if $\sigma = 0.5$? or $\sigma = 2$? Give intuition for your answers.

3. **Unemployment fluctuations in a discrete time search model.** Let time be $t = 0, 1, 2, ...$ Risk neutral workers and firms have common time discount factor β and are matched according to a Cobb-Douglas matching function

$$F(u_t, v_t) = \bar{m}\, u_t^{1-\alpha} v_t^{\alpha}, \qquad \bar{m} > 0, \qquad 0 < \alpha < 1$$

where u_t denotes the unemployment rate and v_t the vacancy rate at time t. Let $\theta_t \equiv v_t/u_t$ denote labor market tightness and let $f(\theta_t)$ and $q(\theta_t)$ denote the job finding probability and vacancy filling probability respectively. If a match forms in period t then the worker and firm are able to start producing in period $t+1$. Within a match, the firm's productivity $z_t > 0$ follows an exogenous stochastic process with long-run mean $\bar{z} > 0$. The worker receives a wage w_t and the firm makes profits of $z_t - w_t$. Job matches are exogenously destroyed with probability $\delta \in (0, 1)$ per period. Firms can create jobs by posting vacancies with a per period cost $\kappa\bar{z} > 0$. When unemployed, workers receive constant flow utility $b\bar{z} \leq w_t$ from unemployment benefits.

The aggregate unemployment rate evolves according to

$$u_{t+1} - u_t = \delta(1 - u_t) - f(\theta_t)u_t$$

given some initial unemployment rate $u_0 > 0$.

(a) Let V_t and J_t denote the value to a firm of a vacancy and a filled job. These satisfy the discrete time Bellman equations

$$V_t = -\kappa\bar{z} + \beta\mathbb{E}_t\{\, q(\theta_t)J_{t+1} + (1 - q(\theta_t))V_{t+1}\,\}$$

$$J_t = z_t - w_t + \beta\mathbb{E}_t\{\,\delta V_{t+1} + (1-\delta)J_{t+1}\,\}$$

Similarly let U_t and W_t denote the value to a worker of unemployment and employment. These satisfy the discrete time Bellman equations

$$U_t = b\bar{z} + \beta\mathbb{E}_t\{\, f(\theta_t)W_{t+1} + (1 - f(\theta_t))U_{t+1}\,\}$$

$$W_t = w_t + \beta\mathbb{E}_t\{\,\delta U_{t+1} + (1-\delta)W_{t+1}\,\}$$

Provide an intuitive interpretation of these four Bellman equation.

Now suppose the wage is determined by Nash-Bargaining so that in equilibrium the worker's surplus is a constant fraction $\lambda \in (0,1)$ of the total match surplus

$$W_t - U_t = \lambda S_t, \qquad S_t \equiv W_t - U_t + J_t - V_t$$

Suppose also that free-entry drives the value of a vacancy to $V_t = 0$.

(b) Explain how to solve for the non-stochastic steady state values of labor market tightness, wages, the unemployment rate and the vacancy rate in this setting. Explain intuitively the effects of a change in average productivity $\bar{z}$.

Now suppose that productivity follows a stationary AR(1) process in logs

$$\log z_{t+1} = (1-\phi)\log\bar{z} + \phi\log z_t + \varepsilon_{t+1}, \qquad 0 < \phi < 1$$

where the innovations ε_t are IID $N(0, \sigma_\varepsilon^2)$.

Let the parameter values be $\alpha = 0.5$, $\beta = 1/1.02$, $\delta = 0.04$, $\phi = 0.95$, $\sigma_\varepsilon = 0.01$, $\bar{z} = 1$, $\bar{m} = 0.5$, $b = 0.4$, $\lambda = 0.5$, and $\kappa = 0.28$.

(c) Solve for the non-stochastic steady state values of labor market tightness, wages, the unemployment rate and the vacancy rate.

(d) Use Dynare to solve the model. Suppose the economy is at steady state and that at $t = 0$ there is a 1% innovation to productivity, i.e., $\varepsilon_0 = 0.01$. Use Dynare to calculate and plot the impulse response functions for the log-deviations of labor market tightness, wages, the unemployment rate and the vacancy rate for $T = 100$ periods after the shock.

(e) Simulate a sequence of productivity $\hat{z}_t$ of length $T = 500$ and use this to generate simulated sequences of labor market tightness, wages, the unemployment rate and the vacancy rate. Which of these variables move most closely together? Which is most volatile? Explain.