Advanced Macroeconomics: Problem Set #2

Due Wednesday May 1 in class

1. **Automation in a growth model.** Suppose a final good Y is produced by *perfectly competitive* firms using a Cobb-Douglas bundle of tasks

$$Y_t = \exp\left(\int_{N-1}^{N} \log y_t(i)\, di\right)$$

for some given interval $[N-1, N]$. All tasks can be done by labor, but some tasks can be done by labor or capital. In particular, there is a threshold task I such that the production technology for tasks $i > I$ is

$$y_t(i) = a_l l_t(i), \qquad i > I$$

while the production technology for tasks $i \leq I$ is

$$y_t(i) = a_k k_t(i) + a_l l_t(i), \qquad i \leq I$$

Each task is produced under perfectly competitive conditions taking as given the wage rate W_t and the rental rate R_t. To simplify the analysis, we tentatively suppose that W_t, R_t are such that

$$\frac{R_t}{a_k} < \frac{W_t}{a_l} \tag{$*$}$$

There are L identical households each of which supplies one unit of labor and seeks to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1$$

subject to

$$c_t + k_{t+1} = W_t + (R_t + 1 - \delta)k_t \qquad 0 < \delta < 1$$

Let $C_t = c_t L$, $K_t = k_t L$ and L denote aggregate consumption, capital, and labor. In equilibrium the factor markets clear with $K_t = \int k_t(i)\, di$ and $L = \int l_t(i)\, di$.

   (a) Let $Y = F(K, L)$ denote the aggregate production function, i.e., the amount of final output that the economy produces with aggregate capital K and labor L. Derive the aggregate production function for this economy.

   (b) Show that in order for condition $(*)$ to hold the aggregate capital stock K_t must exceed a certain threshold

$$K_t > \hat{K}$$

   Provide a formula for this threshold $\hat{K}$ in terms of the underlying parameters of the model.

(c) Solve for the steady state values of aggregate consumption, capital, output and the wage and rental rate in terms of model parameters. Is condition $(*)$ always satisfied in steady state? Explain.

(d) Let the parameter values be $N = 1$, $L = 1$, $a_l = 0.1$, $a_k = 0.2$, $\beta = 1/1.05$, $\delta = 0.05$. For each of the following grid of values

$$I \in \{0.25, 0.26, 0.27, ..., 0.49, 0.50\}$$

calculate and plot the steady state values of aggregate consumption, capital, output and wages. Does more automation increase output? Does more automation decrease wages? What is the role of capital accumulation? Explain your findings.

(e) How if at all do your answers to part (d) change if $\beta = 1/1.03$? Or if $\beta = 1/1.01$? Explain.

2. **Markups in a business cycle model.** Consider a real business cycle model where L identical households seek to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big(\log c_t - \frac{l_t^{1+\varphi}}{1+\varphi} \Big), \qquad 0 < \beta < 1, \quad \varphi > 0$$

subject to the budget constraints

$$c_t + k_{t+1} = W_t l_t + (R_t + 1 - \delta) k_t + \pi_t, \qquad 0 < \delta < 1$$

where π_t denotes lump-sum profits paid out by firms.

Final output Y_t is produced by *perfectly competitive* firms using a CES bundle of intermediates

$$Y_t = \left(\int_0^1 y_t(i)^{1/\mu} \, di \right)^{\mu}, \qquad \mu > 1$$

The final good firms buy intermediate goods at prices $p_t(i)$ from intermediate producers $i \in [0, 1]$. The intermediate producers are *monopolistically competitive* and choose prices $p_t(i)$ and output $y_t(i)$ to maximize profits understanding their market power.

Intermediate producers have the Cobb-Douglas production function

$$y_t(i) = z_t k_t(i)^{\alpha} l_t(i)^{1-\alpha}, \qquad 0 < \alpha < 1$$

and take the economy-wide rental rate R_t and wage rate W_t as given. The exogenous stochastic process for productivity z_t is common to all firms.

Let $C_t = c_t L$, $K_t = k_t L$ and $L_t = l_t L$ denote aggregate consumption, capital, and employment. In equilibrium the factor markets clear with $K_t = \int k_t(i) \, di$ and $L_t = \int l_t(i) \, di$.

(a) Let $\mathsf{TC}_t(y)$ denote the total cost function of each intermediate producer. Show that the total cost function is linear in output

$$\mathsf{TC}_t(y) = \mathsf{mc}_t \, y$$

for some marginal cost mc_t. Derive an expression for marginal cost mc_t in terms of the factor prices W_t, R_t and productivity z_t. Show that intermediate producers set prices that are a *markup* over marginal cost and that this markup is equal to the parameter μ.

(b) Now consider a *symmetric equilibrium* where all intermediate producers set the same price $p_t(i) = p_t$. Derive the key conditions that allow you to show how consumption, capital and employment are determined in this equilibrium. Also explain how prices, the wage rate, rental rate of capital, and profits are determined.

(c) Solve for the non-stochastic steady-state values of consumption, capital and employment in terms of model parameters. Solve also for the steady-state values of producer prices, the wage rate, rental rate of capital, and profits.

(d) Suppose the economy is in the steady state you found in (c). Then suddenly there is a permanent increase in producer market power such that the markup increases permanently from μ to $\mu' > \mu$. Explain the long run responses of consumption, capital, employment, the wage rate, rental rate, and profits in response to this permanent rise in markups.

Now suppose productivity and markups follow independent stationary AR(1) processes in logs

$$\log z_{t+1} = (1-\phi_z)\log \bar{z} + \phi_z \log z_t + \varepsilon_{z,t+1}, \qquad 0 < \phi_z < 1$$

where the innovations $\varepsilon_{z,t}$ are IID $N(0, \sigma^2_{\varepsilon,z})$, and

$$\log \mu_{t+1} = (1-\phi_\mu)\log \bar{\mu} + \phi_\mu \log \mu_t + \varepsilon_{\mu,t+1}, \qquad 0 < \phi_\mu < 1$$

where the innovations $\varepsilon_{\mu,t}$ are IID $N(0, \sigma^2_{\varepsilon,\mu})$.

Let the parameter values be $\alpha = 0.3$, $\beta = 1/1.01$, $\delta = 0.02$, $\varphi = 1$, $\bar{z} = 1$, $\bar{\mu} = 1.15$, with common persistence $\phi_z = \phi_\mu = 0.95$ and innovation standard deviations $\sigma_{\varepsilon,z} = \sigma_{\varepsilon,\mu} = 0.01$.

(e) Use Dynare to solve the model. Use Dynare to calculate and plot the impulse response functions for the log-deviations of consumption, investment, output, employment, the wage rate, rental rate, and profits in response to both (i) a one standard deviation productivity shock, and (ii) a one standard deviation markup shock. How do the dynamic responses of the economy to these shocks compare? Give as much intuition as you can for your findings. Compare the dynamics of the economy in response to this transitory markup shock to the long-run effect of a permanent change in markups as in part (d) above.

(f) Use Dynare to calculate the standard deviations and cross-correlations of the log-deviations of consumption, investment, output, employment, the wage rate, rental rate, and profits conditional on (i) only productivity shocks, (ii) only markup shocks, and (iii) both shocks together. Explain your findings.