Advanced Macroeconomics

Lecture 8: growth theory and dynamic optimization, part six

Chris Edmond

1st Semester 2019

This class

- Automation
  - what are the economic consequences of automation?
  - will automation increase or decrease wages?
  - does automation differ from factor-augmenting technical change?

Automation

- Simple task-based model of automation

  [based on Acemoglu-Autor (2011) and others]

- Interval of *tasks* $i \in [N-1, N]$, some of which may be automated

- Final output is Cobb-Douglas aggregate

$$Y = \exp\left(\int_{N-1}^{N} \log y(i)\, di \right)$$

  Limit of CES aggregator as elasticity of substitution $\theta \to 1$

Automation

- All tasks can be done by labor, $l(i)$
- Some tasks can be done by labor, $l(i)$ or capital, $k(i)$
- Order tasks such that labor has *comparative advantage* in higher i
- Let $I \in [N-1, N]$ denote the *threshold task* such that
  - tasks $i > I$ done by labor
  - tasks $i \leq I$ done by labor or capital
- Increases in I mean a larger range of tasks can be automated

Production

- Tasks that can only be done by labor

$$y(i) = A_L\, a_l(i) l(i), \qquad i > I$$

- Tasks that can be done by labor or capital

$$y(i) = A_K\, a_k(i) k(i) + A_L\, a_l(i) l(i), \qquad i \leq I$$

- Conventional *factor-augmenting* productivities A_L, A_K

- Task-specific productivities $a_l(i), a_k(i)$. Tasks ordered such that labor has comparative advantage in higher i means

$$\frac{a_l(i)}{a_k(i)} \qquad \text{is strictly increasing in } i$$

Four notions of technical change

(1) Conventional factor-augmenting productivity growth in A_L or A_K

(2) Automation, increase in I that allows capital to do more tasks

(3) Intensive margin change in $a_l(i)$ or $a_k(i)$ holding I fixed

(4) Task-creating technical change in N

These will have different implications for wages and output

Market structure

- To isolate effect of automation, go back to perfect competition
- Price equals marginal cost

$$p(i) = c(i)$$

- Fixed supplies of labor L and capital K, market clearing

$$L = \int l(i)\, di$$

$$K = \int k(i)\, di$$

(some factor demands $l(i)$ and $k(i)$ will be zero)

Marginal cost

- Let W denote the wage rate. For tasks only done by labor, marginal cost is

$$c(i) = \frac{W}{A_L a_l(i)}, \qquad i > I$$

- Let R denote the rental rate. For tasks done by labor or capital, marginal cost depends on which is cheaper

$$c(i) = \min\left[\,\frac{R}{A_K a_k(i)}\,,\,\frac{W}{A_L a_l(i)}\,\right], \qquad i \leq I$$

Simplifying assumption

- Assume that for the threshold task

$$\frac{R}{A_K a_k(I)} < \frac{W}{A_L a_l(I)} \qquad (*)$$

  so that

$$c(i) = \frac{R}{A_K a_k(i)}, \qquad i \leq I$$

- Tasks $i \leq I$ produced with capital, tasks $i > I$ produced with labor

- We will see the role this assumption plays below

Demand for each task

- Final good producers maximize

$$Y - \int_{N-1}^{N} p(i)y(i)\,di$$

subject to the Cobb-Douglas production function

$$Y = \exp\left(\int_{N-1}^{N} \log y(i)\,di\right)$$

- Implies unit-elastic demand for each task

$$y(i) = \frac{Y}{p(i)}$$

Factor demands

- Tasks $i > I$ produced with labor, so

$$l(i) = \frac{y(i)}{A_L a_l(i)} = \frac{\frac{Y}{p(i)}}{A_L a_l(i)} = \frac{\frac{Y}{c(i)}}{A_L a_l(i)} = \frac{\frac{Y}{\frac{W}{A_L a_l(i)}}}{A_L a_l(i)} = \frac{Y}{W}$$

(and $l(i) = 0$ for $i \leq I$)

- Tasks $i \leq I$ produced with capital, so

$$k(i) = \frac{y(i)}{A_K a_k(i)} = \frac{\frac{Y}{p(i)}}{A_K a_k(i)} = \frac{\frac{Y}{c(i)}}{A_K a_k(i)} = \frac{\frac{Y}{\frac{R}{A_K a_k(i)}}}{A_K a_k(i)} = \frac{Y}{R}$$

(and $k(i) = 0$ for $i > I$)

Factor market clearing

- Factor market clearing then gives

$$L = \int_I^N l(i)\,di = (N - I)\frac{Y}{W}$$

and

$$K = \int_{N-1}^I k(i)\,di = (I - (N-1))\frac{Y}{R}$$

- Hence factor shares are simply

$$s_L \equiv \frac{WY}{L} = N - I$$

and

$$s_K \equiv \frac{RK}{Y} = (I - (N-1))$$

- Increase in I reduces s_L unless N increases by same amount

Aggregate production function

- Output for task $i > I$

$$y(i) = A_L a_l(i) l(i) = A_L a_l(i) \frac{Y}{W} = a_l(i) \frac{A_L L}{s_L}$$

- Output for task $i \leq I$

$$y(i) = A_K a_k(i) k(i) = A_K a_k(i) \frac{Y}{R} = a_k(i) \frac{A_K K}{s_K}$$

Aggregate production function

- Aggregate output is then given by

$$\begin{aligned}\log Y &= \int_{N-1}^{N} \log y(i)\, di \\ &= \int_{N-1}^{I} \log\left(a_k(i)\frac{A_K K}{s_K}\right) di + \int_{I}^{N} \log\left(a_l(i)\frac{A_L L}{s_L}\right) di\end{aligned}$$

- Collecting terms and simplifying we can write this

$$Y = Z\left(\frac{A_K K}{s_K}\right)^{s_K}\left(\frac{A_L L}{s_L}\right)^{s_L}$$

where

$$Z = \exp\left(\int_{N-1}^{I} \log a_k(i)\, di + \int_{I}^{N} \log a_l(i)\, di\right)$$

Factor-augmenting technical change

- Increase in A_L gives

$$\frac{d\log W}{d\log A_L} = \frac{d\log Y/L}{d\log A_L} = s_L > 0$$

- Increase in A_K gives

$$\frac{d\log W}{d\log A_K} = \frac{d\log Y/L}{d\log A_K} = s_K > 0$$

- Factor-augmenting technical change increases wages and output per worker, labor share does not change

- This is the conventional kind of technical change you've seen before

- Similar effects from increases in $a_l(i)$ or $a_k(i)$ holding I and N fixed

Automation: two effects

- Consider increase in I holding N fixed. Larger fraction of tasks can be automated, labor displaced by capital

- By the definition of labor's share we can write

$$\frac{d\log W}{dI} = \frac{d\log Y/L}{dI} + \frac{d\log s_L}{dI}$$

- Two effects on wages: (i) *labor productivity effect*

$$\frac{d\log Y/L}{dI}$$

and (ii) *displacement effect*

$$\frac{d\log s_L}{dI}$$

- Displacement effect is clearly negative

$$\frac{d\log s_L}{dI} = -\frac{1}{N-I} < 0$$

What about the labor productivity effect?

Automation: labor productivity effect

- Some tedious algebra (see appendix below) gives

$$\frac{d\log Y/L}{dI} = \log\left(\frac{W}{A_L a_l(I)}\right) - \log\left(\frac{R}{A_K a_k(I)}\right) > 0$$

  which is positive by assumption $(*)$ above

- Output per worker rises precisely because the marginal task is cheaper to produce with capital

- If that was not the case, increase in I would not in fact displace labor in the first place

Automation: net effect on wages

- Hence net effect on wages is ambiguous

$$\frac{d \log W}{dI} = -\frac{1}{N - I} + \frac{d \log Y/L}{dI}$$

- Automation increases output per worker but reduces wages if increase in output per worker is small relative to displacement

- Automation will reduce wages if labor productivity effect is small

Automation: net effect on wages

- When is labor productivity effect small? When

$$\frac{d \log Y/L}{dI} = \log\left(\frac{W}{A_L a_l(I)}\right) - \log\left(\frac{R}{A_K a_k(I)}\right) \approx 0$$

- That is, when the cost-reduction gains from switching to capital are negligible

- In other words, it is *dramatic* forms of automation that are more likely to lead to net wage gains

New tasks

- Now consider increase in N holding I fixed. Creation of new tasks in which labor has comparative advantage

- By the definition of labor's share we can write

$$\frac{d\log W}{dN} = \frac{d\log Y/L}{dN} + \frac{d\log s_L}{dN}$$

- Latter effect is clearly positive

$$\frac{d\log s_L}{dN} = \frac{1}{N-I} > 0$$

What about the former effect?

New tasks

- Some more tedious algebra (see appendix below) gives

$$\frac{d \log Y/L}{dN} = \log\left(\frac{R}{A_K a_k(N-1)}\right) - \log\left(\frac{W}{A_L a_l(N)}\right) \qquad (**)$$

- This will be positive if the marginal cost of newly created tasks

$$c(N) = \frac{W}{A_L a_l(N)}$$

is lower than the marginal cost of the least-productive automated tasks that are destroyed

$$c(N-1) = \frac{R}{A_K a_k(N-1)}$$

- If (∗∗) satisfied, net effect is that new tasks increase wages

Joint effect of increase in I and N

- Putting these calculations together we get

$$d\log W = \left\{\log\left(\frac{R}{A_K a_k(N-1)}\right) - \log\left(\frac{W}{A_L a_l(N)}\right)\right\} dN$$
$$+ \left\{\log\left(\frac{W}{A_L a_l(I)}\right) - \log\left(\frac{R}{A_K a_k(I)}\right)\right\} dN + \frac{1}{N-I}(dN - dI)$$

- If $dN \approx dI$ so that the labor share is unchanged, this collapses to

$$d\log W = \left\{\log\left(\frac{a_l(N)}{a_l(I)}\right) + \log\left(\frac{a_k(I)}{a_k(N-1)}\right)\right\} dN > 0$$

- In short for wages to rise and labor's share to remain constant we need $dN \approx dI$ so that the creation of tasks in which labor has a comparative advantage balances labor displacement

Next class

- RBC-style *stochastic* growth model
  - exogenous shocks to productivity
  - outcomes are stochastic processes for output, consumption etc
  - underlies most models of economic fluctuations

Appendix: bonus algebra

Effect of I on Y/L

- Differentiating the aggregate production function with respect to I and collecting terms gives

$$\frac{d \log Y/L}{dI} = \frac{d \log Z}{dI} + \log\left(\frac{A_K K}{A_L L}\right) + \log\left(\frac{N-I}{I-N-1}\right)$$

- Now write

$$\log Z = \int_{N-1}^{I} \log a_k(i)\, di + \int_{I}^{N} \log a_l(i)\, di$$

- Hence

$$\frac{d \log Z}{dI} = \log a_k(I) - \log a_l(I)$$

Effect of I on Y/L

- So we have

$$\frac{d\log Y/L}{dI} = \log\left(\frac{a_k(I)}{a_l(I)}\right) + \log\left(\frac{A_K K}{A_L L}\right) + \log\left(\frac{N-I}{I-N-1}\right)$$

- Then recalling the expressions for the factor shares

$$\frac{N-I}{I-N-1} = \frac{WL}{RK}$$

- Hence this simplifies to

$$\frac{d\log Y/L}{dI} = \log\left(\frac{a_k(I)}{a_l(I)}\right) + \log\left(\frac{A_K}{A_L}\right) + \log\left(\frac{W}{R}\right)$$

which is the same as given on slide 33 above, namely

$$\frac{d\log Y/L}{dI} = \log\left(\frac{W}{A_L a_l(I)}\right) - \log\left(\frac{R}{A_K a_k(I)}\right) > 0$$

Effect of N on Y/L

- Similarly, differentiating the aggregate production function with respect to N and collecting terms gives

$$\frac{d \log Y/L}{dN} = \frac{d \log Z}{dN} - \log\left(\frac{A_K K}{A_L L}\right) - \log\left(\frac{N-I}{I-N-1}\right)$$

- Now write

$$\log Z = \int_{N-1}^{I} \log a_k(i)\, di + \int_{I}^{N} \log a_l(i)\, di$$

- Hence

$$\frac{d \log Z}{dN} = -\log a_k(N-1) + \log a_l(N)$$

Effect of N on Y/L

- So we have

$$\frac{d\log Y/L}{dN} = -\log\left(\frac{a_k(N-1)}{a_l(N)}\right) - \log\left(\frac{A_K K}{A_L L}\right) - \log\left(\frac{N-I}{I-N-1}\right)$$

- Then recalling the expressions for the factor shares

$$\frac{N-I}{I-N-1} = \frac{WL}{RK}$$

- Hence this simplifies to

$$\frac{d\log Y/L}{dN} = -\log\left(\frac{a_k(N-1)}{a_l(N)}\right) - \log\left(\frac{A_K}{A_L}\right) - \log\left(\frac{W}{R}\right)$$

which is the same as given on slide 37 above, namely

$$\frac{d\log Y/L}{dN} = \log\left(\frac{R}{A_K a_k(N-1)}\right) - \log\left(\frac{W}{A_L a_l(N)}\right)$$