Advanced Macroeconomics

Lecture 22: financial crises, part two

Chris Edmond

1st Semester 2019

This class

- Financial crises part two
- Heterogeneous beliefs and leverage cycles
- Further reading
 - ◊ Geanakoplos (2009): Leverage cycles, NBER Macro Annual.

Geanakoplos

- Two dates $t \in \{0, 1\}$
- Two states $s \in \{U, D\}$ at date t = 1
- Two commodities
 - (i) consumption good, durable (costlessly storable \rightarrow also risk-free asset)
 - (ii) risky asset, not consumable but state-contingent payoffs, $x_U > x_D$ in units of consumption
- Continuum $h \in [0, 1]$ of agents with *heterogeneous beliefs*

- agents differ in optimism about s = U

Heterogeneous beliefs

• Continuum $h \in [0, 1]$ of agents with heterogeneous beliefs

$$\operatorname{Prob}[s = U \,|\, h] = h$$

$$\operatorname{Prob}[s=D \mid h] = 1 - h$$

- Agent h = 1 is most optimistic about s = U, agent h = 0 is most pessimistic about s = U
- Agents with sufficiently high *h* are *natural buyers* of the asset
- Agents otherwise identical
 - risk neutral expected utility, indifferent to timing of consumption
 - identical initial endowments, each have one unit of each commodity

Heterogeneous beliefs



Agent h = 1 is most optimistic about s = U. Agents with sufficiently high h are *natural buyers* of the asset. Cutoff h^* determined endogenously in equilibrium.

Two-period example



Two states $s \in \{U, D\}$ possible at date t = 1. Asset pays x_U in good state but only $x_D < x_U$ in bad state. Agent $h \in [0, 1]$ assigns subjective probability h to x_U and probability 1 - h to x_D

No borrowing benchmark

• Expected utility

$$u_h = c_0 + h c_U + (1 - h) c_D$$

• Budget constraints if no borrowing

$$c_0 + p y_0 = 1 + p$$

 $c_U = x_U y_0$

 $c_D = x_D y_0$

- Consumption good is numeraire, p and y_0 are relative price of and quantity of risky asset held at date t = 0. All agents have initial endowment of one unit of each commodity
- No short selling, sales of asset limited by endowment (i.e., $y_0 \ge 0$)

Cutoff agent h^*

• Linear objective and constraints, solution typically at a corner. Any agent h such that

 $x_U h + x_D \left(1 - h\right) > p$

expects payoff greater than price, buys as much as possible

• Any agent *h* such that

 $x_U h + x_D \left(1 - h\right) < p$

expects payoff less than price, sells as much as possible

• Cutoff agent has belief $h = h^*$ such that just indifferent

$$x_U h^* + x_D (1 - h^*) = p \qquad \Rightarrow \qquad h^* = \frac{p - x_D}{x_U - x_D}$$

Solving simultaneously for p and h^*

• Asset demands

$$y_0^h = \begin{cases} 0 & h \in [0, h^*) \\ \frac{1+p}{p} & h \in [h^*, 1] \end{cases} \quad \text{where } h^* = \frac{p - x_D}{x_U - x_D} \tag{1}$$

• Market clearing condition for asset

$$1 = \int_0^1 y_0^h \, dh$$

With these asset demands

$$1 = \int_0^1 y_0^h dh = \int_0^{h^*} 0 dh + \int_{h^*}^1 \frac{1+p}{p} dh = \frac{1+p}{p} (1-h^*) \quad (2)$$

• Two equations to solve for p, h^*

Numerical example

• Suppose $x_U = 1, x_D = 0.2$. Then cutoff belief h^*

$$h^* = \frac{p - 0.2}{1 - 0.2} = 1.25p - 0.25$$

• Market clearing

$$1 = \frac{1+p}{p}(1-h^*) = \frac{1+p}{p}(1.25 - 1.25p)$$

• Rearrange to get quadratic in p

$$p^2 + 0.8p - 1 = 0$$

Only positive solution is p = 0.68 which then implies $h^* = 0.60$

Borrowing at exogenous collateral rates

- Suppose borrowing, but constrained by *exogenous collateral rates*
- Loan promises φ are noncontingent, same in every state
- Loan collateral is the asset, which can be seized if default. A promise of φ gives lender

 $\min[\varphi, x_U]$ if s = U, good news

 $\min[\varphi, x_D]$ if s = D, bad news

• Motivates simple exogenous *collateral constraint*

$$\varphi_0 \le x_D y_0$$

Biggest promise that is *sure* to be covered by collateral

Borrowing at exogenous collateral rates

• Expected utility

$$u^{h} = c_0 + h c_U + (1 - h) c_D$$

• Constraints if borrowing at exogenous collateral rate

$$c_0 + p y_0 = 1 + p + \frac{1}{1+r}\varphi_0$$

$$\varphi_0 \le x_D y_0$$

$$c_U = x_U y_0 - \varphi_0$$

 $c_D = x_D y_0 - \varphi_0$

• Borrowing if $\varphi_0 > 0$, lending if $\varphi_0 < 0$, r is interest rate. No borrowing is special case with collateral constraint $\varphi_0 \leq 0y_0$.

Solving the model

• Guess interest rate r = 0 (linear utility, endowments large enough)

• As before, cutoff agent h^* just indifferent

$$x_U h^* + x_D (1 - h^*) = p \qquad \Rightarrow \qquad h^* = \frac{p - x_D}{x_U - x_D}$$

• Agents $h < h^*$ sell as much as possible, $y_0^h = 0$ for all $h < h^*$

• Agents $h > h^*$ buy as much as possible. To do this, borrow the maximum

$$\varphi_0^h = x_D y_0^h$$

and so for these agents

$$y_0^h = \frac{1+p+\varphi_0^h}{p} = \frac{1+p+x_D y_0^h}{p} \quad \Leftrightarrow \quad y_0^h = \frac{1+p}{p-x_D}$$

Solve simultaneously for p, h^* as before

Numerical example

• Suppose again $x_U = 1, x_D = 0.2$. Then market clearing for asset is

$$1 = \int_0^1 y_0^h dh = \int_{h^*}^1 \frac{1+p}{p-0.2} dh = \frac{1+p}{p-0.2}(1-h^*)$$

• Eliminating h^* using the indifference condition for the cutoff agent now gives quadratic

$$p^2 + 0.8p - 1.16 = 0$$

Only positive solution is p = 0.75 which then implies $h^* = 0.69$. Asset prices higher, marginal buyer is more optimistic than without borrowing

Numerical example

• At price p = 0.75, buyers $h > h^* = 0.69$ have

risky asset = $y_0^h = 3.2$, promise = $\varphi_0^h = 0.64$

- Sellers $h < h^*$ have zero asset purchases and lend (from loans market clearing, about $\varphi_0^h = -(1 h^*)0.64/h^* = -0.3$ each)
- The leverage ratio is, in this example,

leverage =
$$\frac{\text{asset value}}{\text{asset value} - \text{debt value}} = \frac{p}{p - 0.2} = \frac{0.75}{0.55} \approx 1.4$$

(of course, all $h < h^*$ are not levered). Equivalently, loan/value ratio is 0.2/0.75 = 27% and margin or haircut is 0.55/0.75 = 73%

Discussion

- Ability to borrow allows most optimistic agents to 'leverage' their beliefs, borrowing to spend more on asset
- Fewer optimistic agents required to buy asset stock, marginal buyer h^* is more optimistic, asset prices higher
- Asset prices don't just depend on payoff fundamentals, but also on lending standards. Loose lending standards ⇒ higher asset prices
- Why? Because asset prices depend on beliefs and beliefs of marginal buyer change as lending standards change (because *who the marginal buyer is* changes)

Leverage cycle: three-period example

- Three dates $t \in \{0, 1, 2\}$
- Binomial tree
 - two states $s_1 \in \{U, D\}$ at date t = 1
 - so four states $s_2 \in \{UU, UD, DU, DD\}$ at date t = 2
- Agent $h \in [0, 1]$ believes upticks occur with probability h
- Risky asset pays off at terminal date t = 2, nothing at t = 1
- But is traded based on interim information s_1 at t = 1

Leverage cycle: three-period example



Two states $s_1 \in \{U, D\}$ at date t = 1. Four states $s_2 \in \{UU, UD, DU, DD\}$ possible at date t = 2. State-contingent payoff at terminal date, nothing at date t = 1. But traded at date t = 1 based on interim information s_1 .

Leverage cycle: three-period example

- Suppose $x_{UU} = x_{UD} = x_{DU} = 1$ but $x_{DD} < 1$
- Then if $s_1 = U$, all uncertainty has been resolved
- Focus on s₁ = D, for which (i) there has been bad news, and
 (ii) there is remaining uncertainty
- Equilibrium characterized in terms of four numbers

 p_0, p_D, h_0^*, h_D^*

asset prices p_0, p_D and cutoff beliefs h_0^*, h_D^* at t = 0 and $s_1 = D$

Cutoff beliefs h_0^*, h_D^*



Initial buyers wiped out if bad news, $s_1 = D$. Risky asset then bought by agents $h \in [h_D^*, h_0^*]$ with less optimistic beliefs.

Numerical example

• Suppose $x_{DD} = 0.2$ as in previous examples. Solution works out to

$$p_0 = 0.95, \quad p_D = 0.69, \quad h_0^* = 0.87, \quad h_D^* = 0.61$$

- Asset price crashes from $p_0 = 0.95$ to $p_D = 0.69$ on bad news
- But bad news alone only explains part of the fall in asset prices
- In addition, marginal buyer is an agent with less optimistic beliefs, initial buyers (most optimistic) wiped out
- Moreover, it becomes harder to borrow (collateral rate falls from $p_D = 0.69$ to $x_{DD} = 0.2$)

Leverage ratios

• Initial leverage

$$\frac{p_0}{p_0 - p_D} = \frac{0.95}{0.95 - 0.69} = 3.65$$

• Falls to

$$\frac{p_D}{p_D - x_{DD}} = \frac{0.69}{0.69 - 0.20} = 1.41$$

- Or equivalently, initial margins (haircuts) rise from 1/3.65 = 27% to 1/1.41 = 71%
- In short, the bad news dramatically tightens borrowing constraints, which amplifies the fall in asset prices