Advanced Macroeconomics

Lecture 20: unemployment, part two

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This class

- More on the search model of unemployment
- Comparative statics and dynamics local to steady state
- Constrained efficiency [not examinable]

Steady state equilibrium

- Steady state (u, w, θ) solves

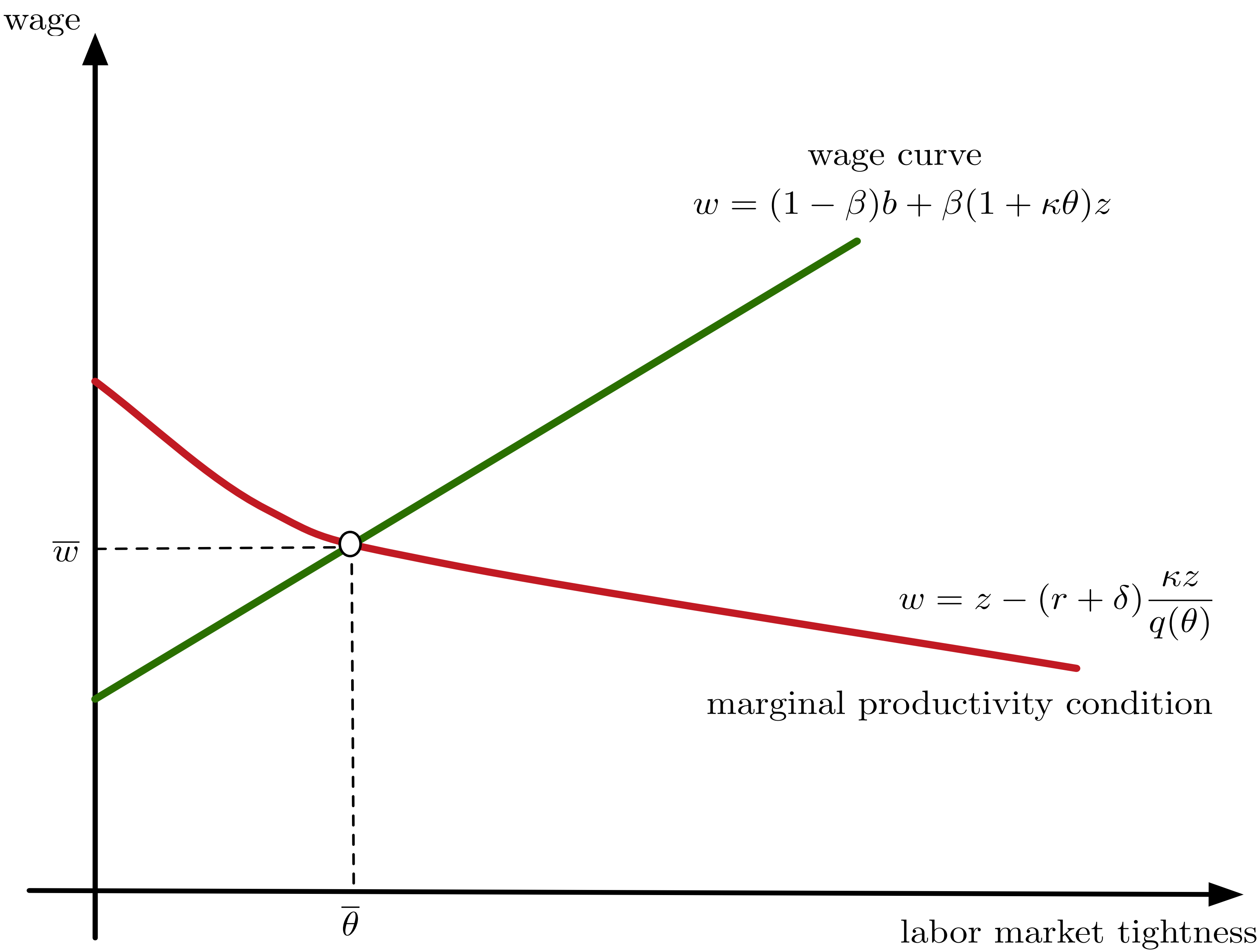
$$(1) \qquad w = (1-\beta)b + \beta(1+\kappa\theta)z \qquad \text{(wage curve)}$$

$$(2) \qquad w = z - (r+\delta)\frac{\kappa z}{q(\theta)} \qquad \text{(marginal productivity)}$$

$$(3) \qquad u = \frac{\delta}{\delta + f(\theta)} \qquad \text{(Beveridge curve)}$$

- Solve (1) and (2) simultaneously for w, θ. Then recover u from (3) and then $v = \theta u$ etc. Recover W, U, J from Bellman equations.

Steady state w, θ



Increase in z

- Consider an increase in productivity z
- Shifts up both wage curve and marginal productivity curve
- So wage w unambiguously rises
- What about net effect on market tightness? To determine this, eliminate w from (1)-(2) to get θ as an implicit function of z

$$g(\theta, z) \equiv (r+\delta) + \beta f(\theta) - (1-\beta)\frac{z-b}{z\kappa}q(\theta) = 0$$

- Then derivative of θ with respect to z is

$$\frac{d\theta}{dz} = -\frac{g_z(\theta, z)}{g_\theta(\theta, z)}$$

where both partial derivatives are evaluated at steady state

Increase in z

- Calculating the partial derivative with respect to z

$$g_z(\theta, z) = -(1-\beta)q(\theta)\frac{b}{\kappa z^2} < 0$$

- Calculating the partial derivative with respect to θ

$$g_\theta(\theta, z) = \beta f'(\theta) - (1-\beta)\frac{z-b}{z\kappa}q'(\theta) > 0$$

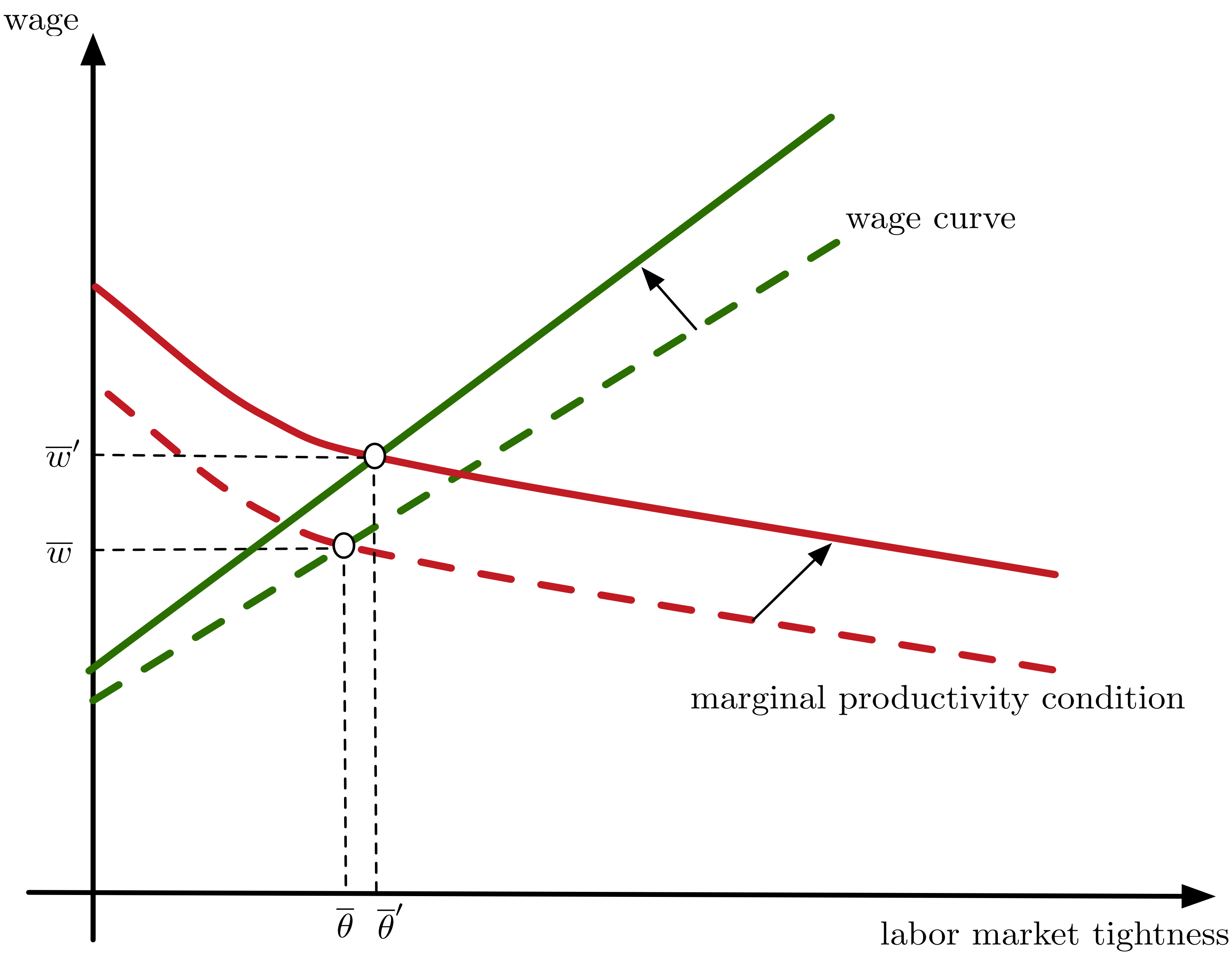
(since $f'(\theta) > 0$ and $q'(\theta) < 0$)

- Hence

$$\frac{d\theta}{dz} = -\frac{g_z(\theta, z)}{g_\theta(\theta, z)} > 0$$

- Shift in marginal productivity dominates shift in wage curve.

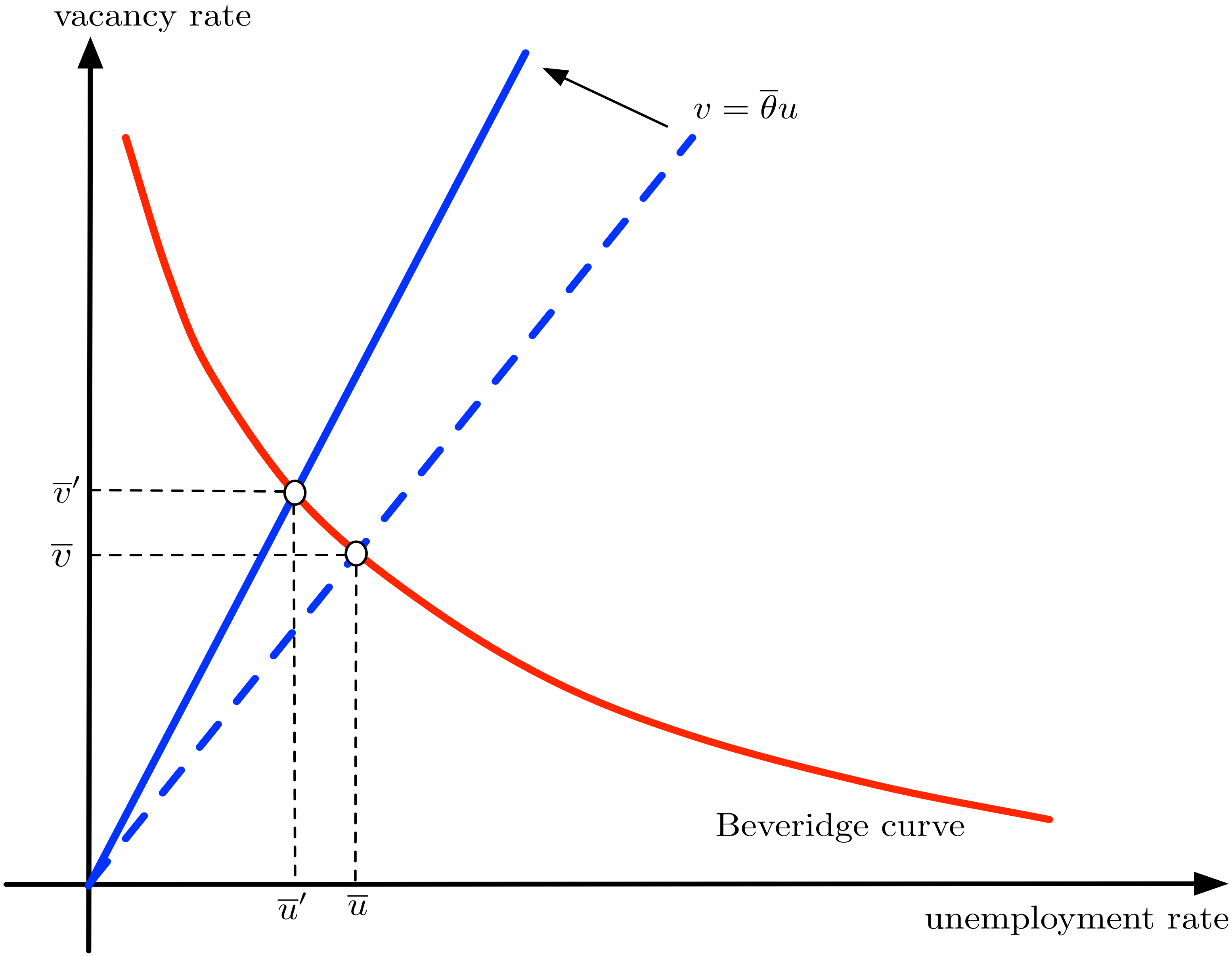
Net effect is for θ to rise



Increase in z

- Increase in z increases w and labor market tightness θ
- Implies counterclockwise rotation along Beveridge curve
- Hence unemployment u falls and vacancies v rise

So u falls and v rises



Aside on balanced growth

- Permanent increase in z permanently reduces u
- Not consistent with balanced growth (implies u trend decreasing)
- Key is that benefits b fixed, so wage does not absorb all of the productivity increase
- If instead $b = \bar{b}z$ for some $\bar{b} \in (0, 1)$ then θ invariant to z hence u, v likewise invariant to z
- If so, have constant unemployment u along balanced growth path

Other comparative statics

- Higher b (or $\bar{b}$) shifts up wage curve
  - w rises but θ falls, hence from Beveridge curve u rises and v falls
- Higher β shifts up wage curve
  - w rises but θ falls, hence from Beveridge curve u rises and v falls
- Higher r shifts down marginal productivity curve
  - w, θ both fall, hence from from Beveridge curve u rises and v falls
- Higher δ shifts down marginal productivity curve
  - w, θ both fall
  - but also shifts out Beveridge curve
  - hence u falls further, net effect on v ambiguous

Dynamics: setup

- Now consider transitional dynamics around steady state
- Stationary environment with constant z, r, δ etc
- Bellman equations for firms

$$rJ(t) = z - w(t) + \dot{J}(t) + \delta(V(t) - J(t))$$

$$rV(t) = -\kappa z + \dot{V}(t) + q(\theta(t))(J(t) - V(t))$$

with change in 'asset values' $\dot{J}(t), \dot{V}(t)$

Dynamics: setup

- Bellman equations for workers

$$rW(t) = w(t) + \dot{W}(t) + \delta(U(t) - W(t))$$

$$rU(t) = b + \dot{U}(t) + f(\theta(t))(W(t) - U(t))$$

- Free entry

$$V(t) = 0 \qquad \Rightarrow \qquad \dot{V}(t) = 0$$

- Nash bargaining

$$W(t) = U(t) + \beta(W(t) - U(t) + J(t))$$

which also implies

$$\dot{W}(t) = \dot{U}(t) + \beta(\dot{W}(t) - \dot{U}(t) + \dot{J}(t))$$

Dynamics: setup

- Same algebra as previous lecture then gives wage curve

$$w(t) = (1-\beta)b + \beta(1+\kappa\theta(t))z$$

- And also have law of motion for unemployment

$$\dot{u}(t) = \delta(1-u(t)) - f(\theta(t))u(t), \qquad u(0) > 0 \text{ given}$$

Dynamics: solution overview

- Now consider dynamics of key variables $u(t), w(t), \theta(t)$
- Unemployment $u(t)$ is predetermined, has initial condition $u(0)$
- Wage $w(t)$ and vacancies $v(t)$ are jump (control) variables
- Hence $\theta(t) \equiv v(t)/u(t)$ is also a jump variable
- Reduces to a single differential equation in $\theta(t)$

Differential equation in $\theta(t)$

- Start with the dynamics of the value of a filled job

$$\dot{J}(t) = (r+\delta)J(t) - (z - w(t))$$

- Using wage curve this becomes

$$\dot{J}(t) = (r+\delta)J(t) + \beta\kappa z\theta(t) - (1-\beta)(z-b)$$

- But from free entry $V(t) = 0$ etc also have

$$J(t) = \frac{\kappa z}{q(\theta(t))}$$

- So J is strictly increasing function of θ

- Can write this as an autonomous differential equation in $\theta(t)$

Differential equation in $\theta(t)$

- Differentiating the last expression

$$\dot{J}(t) = -\kappa z q(\theta(t))^{-2} q'(\theta(t))\dot{\theta}(t)$$

- Eliminating $\dot{J}(t)$ and rearranging gives

$$\boxed{\alpha(\theta(t))\,\frac{\dot{\theta}(t)}{\theta(t)} = g(\theta(t))}$$

where $\alpha(\theta)$ is the elasticity of the vacancy filling rate

$$\alpha(\theta) \equiv -\frac{q'(\theta)\theta}{q(\theta)} \in (0,1)$$

and where $g(\theta)$ is the expression we had on slide 5 above

$$g(\theta) \equiv (r+\delta) + \beta f(\theta) - (1-\beta)\frac{z-b}{z\kappa}q(\theta)$$

Qualitative dynamics

- Since $g(\overline{\theta}) = 0$ and $g'(\theta) > 0$ for all θ we have

$$g(\theta) > 0 \qquad \Leftrightarrow \qquad \theta > \overline{\theta}$$

- And that implies

$$\dot{\theta}(t) > 0 \qquad \Leftrightarrow \qquad \theta(t) > \overline{\theta}$$

- In short, the differential equation in $\theta(t)$ is unstable

- As usual, solution is for $\theta(t)$ to immediately jump to steady state $\overline{\theta}$ so as to avoid explosive trajectories

Qualitative dynamics

- Then have from wage equation that $w(t)$ immediately jumps to

$$\overline{w} = (1-\beta)b + \beta(1+\kappa\overline{\theta})z$$

- And then unemployment evolves essentially independently

$$\dot{u}(t) = \delta(1-u(t)) - f(\overline{\theta})u(t), \qquad u(0) > 0 \text{ given}$$

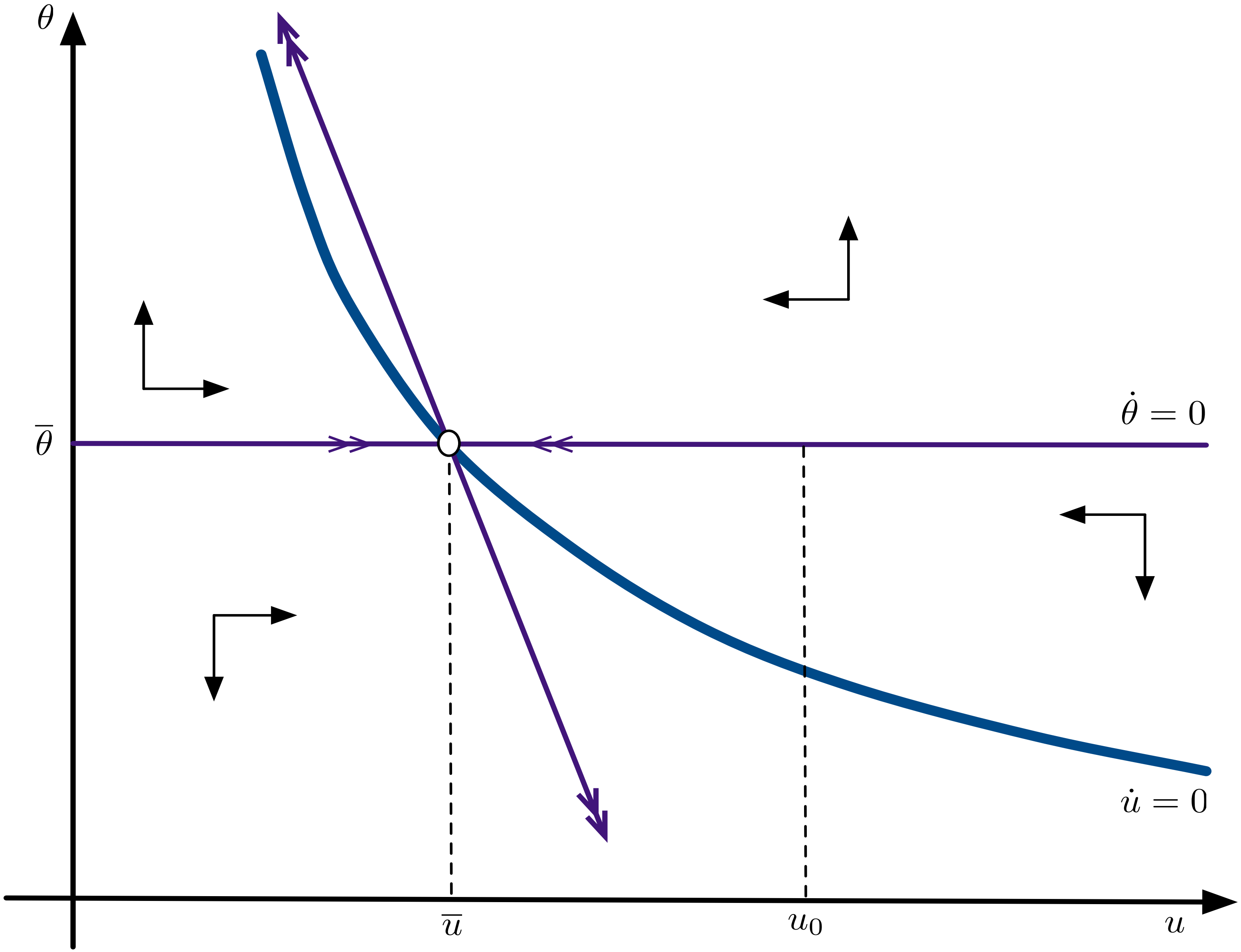
- This can be rewritten

$$\dot{u}(t) = -(\delta + f(\overline{\theta}))(u(t) - \overline{u}), \qquad \bar{u} = \frac{\delta}{\delta + f(\overline{\theta})}$$

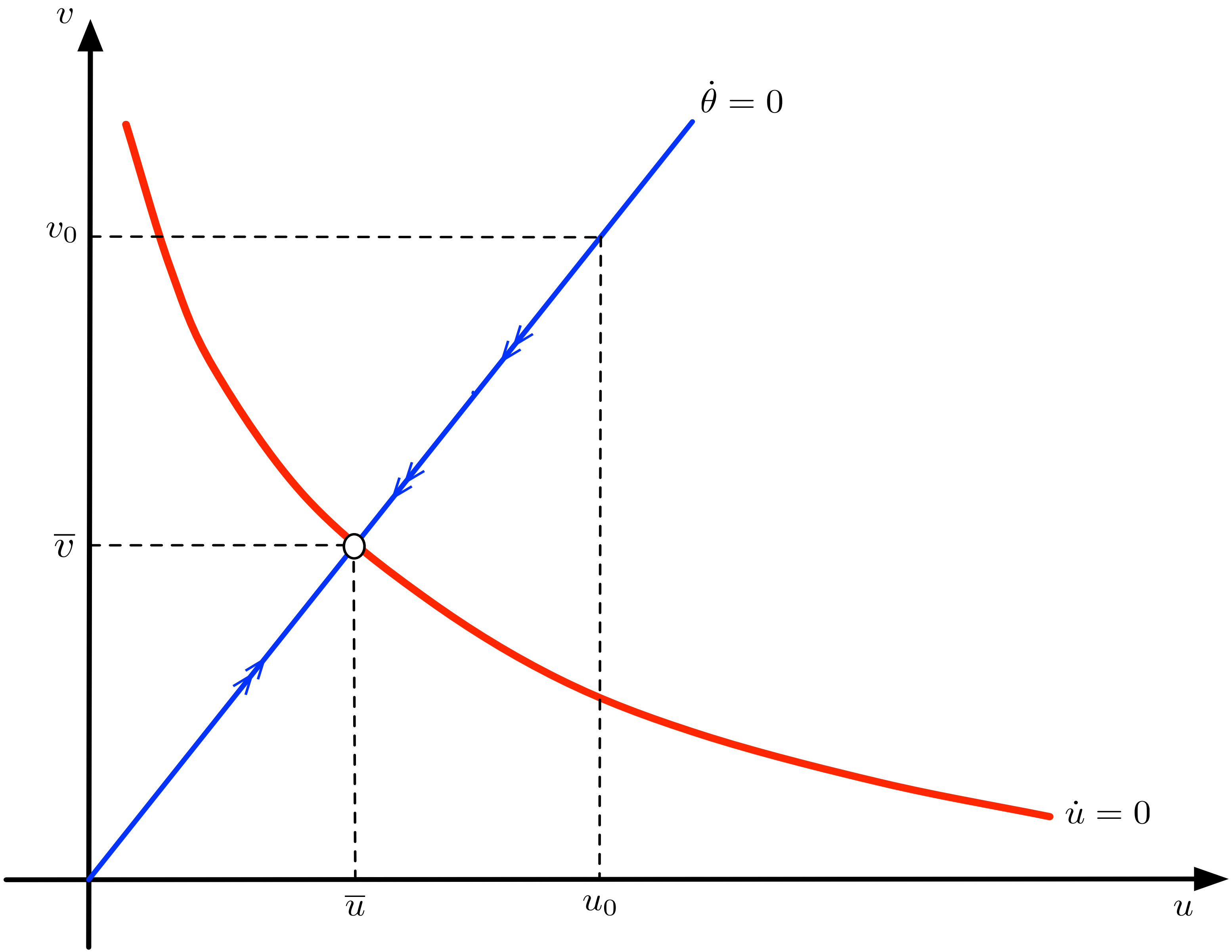
- Hence

$$\dot{u}(t) > 0 \qquad \Leftrightarrow \qquad u(t) < \overline{u}$$

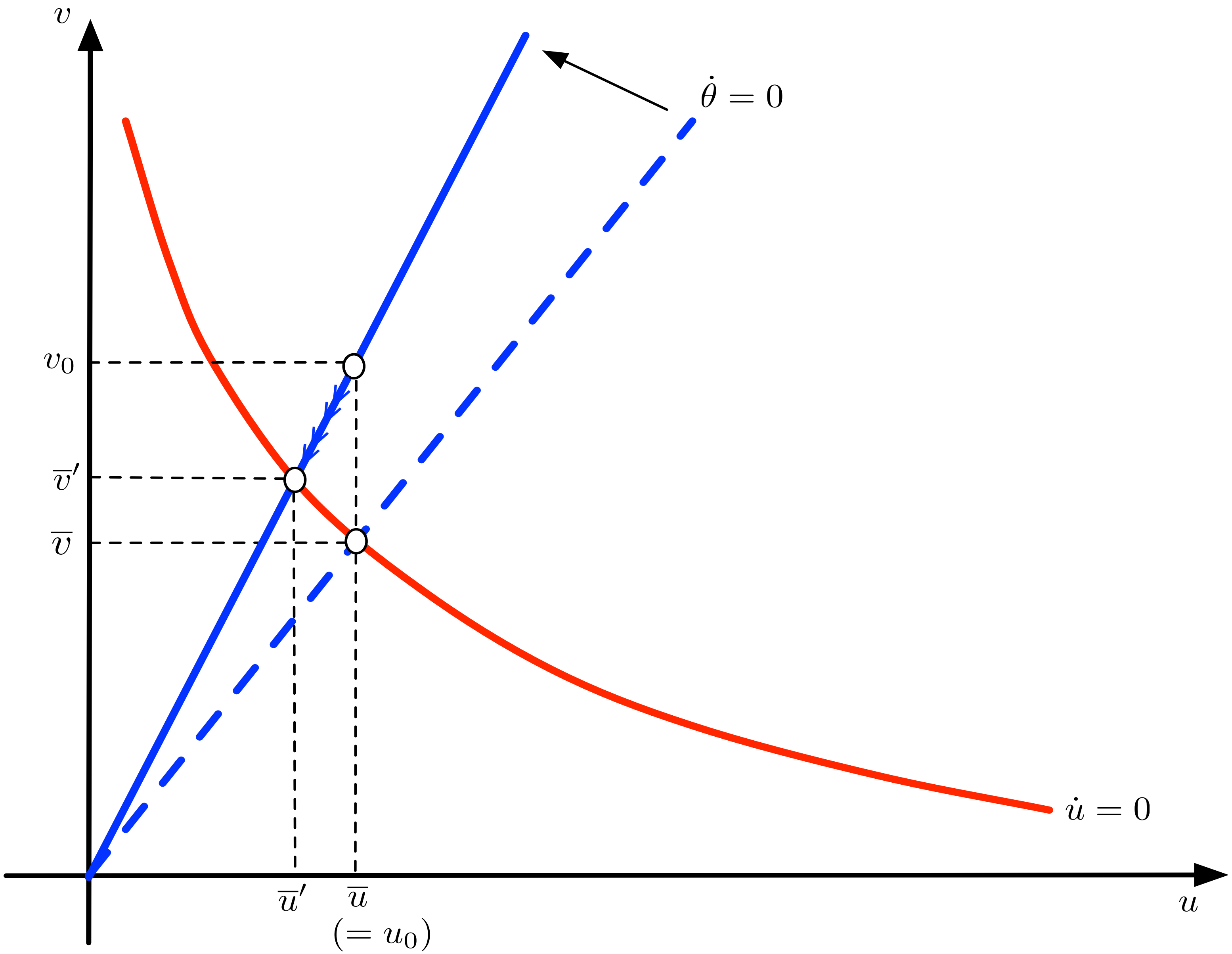
Saddle path dynamics



$$v(t) = \overline{\theta}u(t)$$



Transitional dynamics following increase in z



Appendix: constrained efficiency
[not examinable]

Constrained efficiency

- What is the *efficient* amount of unemployment in this economy? That is, what level of unemployment would be chosen by a planner who faces the same search frictions?

- Planning problem is to choose $u(t), v(t)$ to maximize

$$\int_0^\infty e^{-rt}\left[z(1-u(t)) + bu(t) - \kappa z v(t)\right] dt$$

subject to

$$\dot{u}(t) = \delta(1-u(t)) - f(\theta(t))u(t), \qquad u(0) > 0 \text{ given}$$

$$v(t) = \theta(t)u(t)$$

Constrained efficiency

- Hamiltonian for this problem can be written

$$\mathcal{H} = z(1-u) + bu - \kappa z\theta u + \mu(\delta(1-u) - f(\theta)u)$$

with multiplier μ on the law of motion for unemployment

- Key optimality conditions

$$\theta(t): \qquad -\kappa z u(t) - \mu(t) f'(\theta(t)) u(t) = 0$$

and

$$u(t): \qquad -z + b - \kappa z\theta(t) - \mu(t)(\delta + f(\theta(t))) = r\mu(t) - \dot{\mu}(t)$$

Constrained efficiency

- Evaluating these at steady state and using $q(\theta) = \theta f(\theta)$ these reduce to a single condition determining the planner's optimal θ

$$(r+\delta) + \alpha(\theta) f(\theta) - (1-\alpha(\theta))\frac{z-b}{z\kappa}q(\theta) = 0, \qquad \alpha(\theta) \equiv -\frac{q'(\theta)\theta}{q(\theta)}$$

- By contrast, in the decentralized equilibrium we had

$$(r+\delta) + \beta f(\theta) - (1-\beta)\frac{z-b}{z\kappa}q(\theta) = 0$$

- These two expressions will coincide if and only if

$$\alpha(\theta) = \beta$$

that is, if the elasticity of the vacancy filling rate happens to equal labor's bargaining weight (Hosios 1990).

Cobb-Douglas example

- In general $\alpha(\theta)$ endogenous, but if Cobb-Douglas matching

$$F(u, v) = u^{\alpha} v^{1-\alpha}$$

then $q(\theta) = \theta^{-\alpha}$ and $\alpha(\theta) = \alpha$ is constant

- Then the level of labor market tightness θ and hence the level of unemployment u will be efficient only in the knife edge case

$$\alpha = \beta$$

- Moreover

  – if $\alpha > \beta$, then equilibrium unemployment less than optimal
  – if $\alpha < \beta$, then equilibrium unemployment more than optimal

Intuition: congestion externalities

- Firms and workers congest each other
  - one more hiring firm is good for searching workers but bad for other hiring firms
  - one more searching worker is good for hiring firms but bad for other searching workers
- When $\alpha(\theta) > \beta$, inefficiently many hiring firms and inefficiently low unemployment rate
- When $\alpha(\theta) < \beta$, inefficiently few hiring firms and inefficiently high unemployment rate
- When $\alpha(\theta) = \beta$, externalities are internalized