This class

- First of two lectures on unemployment
- Mortensen-Pissarides model of search unemployment and labor market flows
Motivation

• RBC and new Keynesian models imply fluctuations in employment

• But no unemployment as such

• For that, we need some notion of frictions in the labor market

• Popular approach is to make use of search frictions
Search frictions

• In a standard labor market model
  – firm can hire as much labor as it wants at prevailing wage
  – workers can find employment at prevailing wage

• In search model, neither of these is immediately true
  – unemployed workers need to find jobs
  – firms with vacancies need to find workers

and these activities take *time and resources*

• Of course, search frictions not the only reason for unemployment
Search models of the labor market

- Tractable alternative to labor supply/demand models
- Emphasizes labor market flows (e.g., transitions in/out employment, in/out labor force etc)
- Natural connection to data on job creation and job destruction
Unemployment dynamics

• Simple model of unemployment flows

\[ \dot{u}(t) = \delta(1 - u(t)) - fu(t) \]

with constant job destruction rate \( \delta \) and job finding rate \( f \)

• In steady state, unemployment rate is

\[ \bar{u} = \frac{\delta}{\delta + f} \]

• Transitional dynamics from some initial \( u(0) \)

\[ u(t) = \bar{u} + e^{-\lambda t}(u(0) - \bar{u}), \quad \lambda = \delta + f \]
Matching function

• Let $L > 0$ denote size of the labor force

• Let $m_L$ denote number of job matches, $u_L$ number of unemployed, and $v_L$ number of vacant jobs

• Assume number matches given by matching function

$$m_L = F(u_L, v_L)$$

that is increasing, concave and has constant returns to scale so that

$$m = F(u, v)$$
Matching function

- Job finding rate $f$

  \[ f u = m = F(u, v) \quad \Rightarrow \quad f = \frac{F(u, v)}{u} \]

- Other side of this is vacancy filling rate $q$

  \[ q v = m = F(u, v), \quad \Rightarrow \quad q = \frac{F(u, v)}{v} = \frac{f u}{v} \]

- With constant returns to scale

  \[ f = F(1, \frac{v}{u}) \equiv f(\theta) \]

  \[ q = F\left(\frac{u}{v}, 1\right) \equiv q(\theta) = f(\theta)/\theta \]

  where $\theta \equiv v/u$ is known as ‘labor market tightness’
Matching function

- Job finding rate $f(\theta)$, increasing in labor market tightness. Expected duration unemployment $1/f(\theta)$, decreasing in $\theta$

- Vacancy filling rate $q(\theta)$, decreasing in labor market tightness. Expected duration vacancy $1/q(\theta)$, increasing in $\theta$

- **Example:** if $F(u, v) = u^\alpha v^{1-\alpha}$ for $0 < \alpha < 1$ then

$$f(\theta) = \theta^{1-\alpha}, \quad q(\theta) = \theta^{-\alpha}$$
Estimated relationship between job finding $f$ and labor market tightness $\theta$ in postwar US data. Both expressed as log deviations from trend. A Cobb-Douglas matching function implies a \textit{linear} relationship between $\log f$ and $\log \theta$. 

Estimated matching function
Beveridge curve

- Now write steady state unemployment condition

\[
\begin{align*}
    u &= \frac{\delta}{\delta + f(\theta)}, \\
    \theta &= v/u
\end{align*}
\]  

(1)

- Set of \((v, u)\) satisfying (1) is known as the ‘Beveridge curve’

- An inverse relationship between \(v\) and \(u\). Shifted by changes in the job destruction rate \(\delta\) or the matching technology \(f(\cdot)\)

- Example: if \(F(u, v) = a u^\alpha v^{1-\alpha}\) for \(0 < \alpha < 1\) and \(a > 0\) then

\[
v = \left( \left( \frac{\delta}{a} \right) \left( \frac{1-u}{u^\alpha} \right) \right)^{1/(1-\alpha)}
\]
Beveridge curve

vacancy rate

boom, tight labor market

slump, slack labor market

unemployment rate
Shifts in the Beveridge curve

Outward shift in Beveridge curve (less efficient labor market)
Estimated Beveridge curve

Estimated relationship between job vacancies $v$ and unemployment $u$ in postwar US data. Both expressed as log deviations from trend. This negative relationship is known as the Beveridge curve.
Pissarides (1985)

- Benchmark model of labor market flows
- Continuous time $t \geq 0$, will focus on steady states for now
- Risk neutral workers and firms, constant discount rate $r > 0$
- Unemployed workers and firms with vacancies matched via $F(u, v)$
- Free-entry into vacancy creation
Job creation and destruction

- Firms can employ one worker
- Flow value of production $z > 0$
- Flow wage paid to workers $w$
- Jobs destroyed at exogenous rate $\delta > 0$
- Jobs created by posting vacancies, flow cost $\kappa z > 0$
- Vacancy filling rate $q(\theta)$
Job creation

- Let $J$ denote the present discounted value of a filled job. In steady state this satisfies the Bellman equation

$$rJ = z - w + \delta(V - J), \quad \Rightarrow \quad J = \frac{z - w}{r + \delta} + \frac{\delta}{r + \delta}V$$

- Let $V$ denote the present discounted value of a vacancy

$$rV = -z\kappa + q(\theta)(J - V)$$

- Free-entry into job creation implies $V = 0$, so

$$J = \frac{z\kappa}{q(\theta)}$$

- Together these imply the ‘marginal productivity’ condition

$$w = z - (r + \delta)\frac{z\kappa}{q(\theta)}$$ (2)

- For given wage $w$, this will determine labor market tightness $\theta$. 17
Workers

- Let $W$ denote the present discounted value of being employed

\[ rW = w + \delta(U - W), \quad \Rightarrow \quad W = \frac{w}{r + \delta} + \frac{\delta}{r + \delta}U \]

- Let $U$ denote the present discounted value of being unemployed

\[ rU = b + f(\theta)(W - U) \]

where $b \leq w$ denotes flow value of unemployment benefits etc
Wage determination

- Match between unemployed worker and firm with vacancy creates a mutual profit opportunity. How should these profits be split?

- Flow payments $z - w$ to firm, $w$ to worker

- Wage $w$ determined by *bargaining* between worker and firm

- Choice of $w$ affects job value to individual firm $J(w)$ and to individual worker $W(w)$ taking as given aggregate market conditions $U, V$ etc

- At a wage of $w$, the firm’s surplus from a match is $J(w) - V$ and the worker’s surplus is $W(w) - U$
Generalized Nash bargaining

• Wage $w$ maximizes the *Nash product*

\[
(W(w) - U)^\beta (J(w) - V)^{1-\beta}, \quad 0 \leq \beta \leq 1
\]

where the parameter $\beta$ denotes the workers’ *bargaining power*

• First order condition for this problem can be written

\[
\beta \frac{W'(w)}{W(w) - U} = -(1 - \beta) \frac{J'(w)}{J(w) - V}
\]

Now note that, treating aggregate $U, V$ as given

\[
W'(w) = \frac{1}{r + \delta}, \quad J'(w) = -\frac{1}{r + \delta} = -W'(w)
\]

• So we can write

\[
W = U + \beta S
\]

where $S = W - U + J$ is the total match surplus (given $V = 0$)
Wages and the value of unemployment

- Recall that
  \[ W = \frac{w + \delta U}{r + \delta}, \quad \text{and} \quad J = \frac{z - w}{r + \delta} \]

- Then given surplus splitting \( W - U = \beta(W - U + J) \) we have
  \[ \frac{w}{r + \delta} - \frac{r}{r + \delta} U = \beta \left( \frac{w}{r + \delta} - \frac{r}{r + \delta} U + \frac{z - w}{r + \delta} \right) \]

- Hence in current value terms
  \[ w - rU = \beta(w - rU + z - w) \]

which implies
\[ w = rU + \beta(z - rU) \]
Wage curve

• Hence

\[ w = (1 - \beta)rU + \beta z \]

• Which by the Bellman equation for \( U \) is also

\[ w = (1 - \beta)[b + f(\theta)(W - U)] + \beta z \]

• But worker surplus proportional to firm surplus which is pinned down by free entry

\[ W - U = \frac{\beta}{1 - \beta}J = \frac{\beta}{1 - \beta} \frac{z\kappa}{q(\theta)} \]

• With a bit more algebra and using \( f(\theta)/q(\theta) = \theta \) we get

\[ w = (1 - \beta)b + \beta(1 + \kappa \theta)z \]

which is known as the ‘wage curve’
Steady state equilibrium

- Steady state \((u, w, \theta)\) solves

\[
\begin{align*}
(1) & \quad w = (1 - \beta)b + \beta(1 + \kappa \theta)z & \text{(wage curve)} \\
(2) & \quad w = z - (r + \delta) \frac{\kappa z}{q(\theta)} & \text{(marginal productivity)} \\
(3) & \quad u = \frac{\delta}{\delta + f(\theta)} & \text{(Beveridge curve)}
\end{align*}
\]

- Solve (1) and (2) simultaneously for \(w, \theta\). Then recover \(u\) from (3) and then \(v = \theta u\) etc. Recover \(W, U, J\) from Bellman equations.
Steady state \( w, \theta \)

wage curve
\[
w = (1 - \beta)b + \beta(1 + \kappa \theta)z
\]

marginal productivity condition
\[
w = z - (r + \delta) \frac{\kappa z}{q(\theta)}
\]
Recover $u$ from Beveridge curve

$u = \frac{\delta}{\delta + f(v/u)}$

Beveridge curve

$u = \overline{\theta} u$

vacancy rate