Advanced Macroeconomics

Lecture 19: unemployment, part one

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This class

- First of two lectures on unemployment
- Mortensen-Pissarides model of search unemployment and labor market flows

Motivation

- RBC and new Keynesian models imply fluctuations in *employment*
- But no unemployment as such
- For that, we need some notion of frictions in the labor market
- Popular approach is to make use of *search frictions*

Search frictions

- In a standard labor market model
  - firm can hire as much labor as it wants at prevailing wage
  - workers can find employment at prevailing wage
- In search model, neither of these is immediately true
  - unemployed workers need to find jobs
  - firms with vacancies need to find workers

  and these activities take *time and resources*
- Of course, search frictions not the only reason for unemployment

Search models of the labor market

- Tractable alternative to labor supply/demand models

- Emphasizes *labor market flows*
(e.g., transitions in/out employment, in/out labor force etc)

- Natural connection to data on *job creation* and *job destruction*

Unemployment dynamics

- Simple model of unemployment flows

$$\dot{u}(t) = \delta(1 - u(t)) - fu(t)$$

with constant job destruction rate δ and job finding rate f

- In steady state, unemployment rate is

$$\bar{u} = \frac{\delta}{\delta + f}$$

- Transitional dynamics from some initial $u(0)$

$$u(t) = \bar{u} + e^{-\lambda t}(u(0) - \bar{u}), \qquad \lambda = \delta + f$$

Matching function

- Let $L > 0$ denote size of the labor force
- Let mL denote number of job *matches*, uL number of unemployed, and vL number of vacant jobs
- Assume number matches given by *matching function*

$$mL = F(\, uL \,,\, vL \,)$$

that is increasing, concave and has *constant returns to scale* so that

$$m = F(u \,,\, v)$$

Matching function

- Job finding rate f

$$fu = m = F(u, v) \qquad \Rightarrow \qquad f = \frac{F(u, v)}{u}$$

- Other side of this is vacancy filling rate q

$$qv = m = F(u, v), \qquad \Rightarrow \qquad q = \frac{F(u, v)}{v} = \frac{f\,u}{v}$$

- With constant returns to scale

$$f = F\big(1, \frac{v}{u}\big) \equiv f(\theta)$$

$$q = F\big(\frac{u}{v}, 1\big) \equiv q(\theta) = f(\theta)/\theta$$

where $\theta \equiv v/u$ is known as '*labor market tightness*'

Matching function

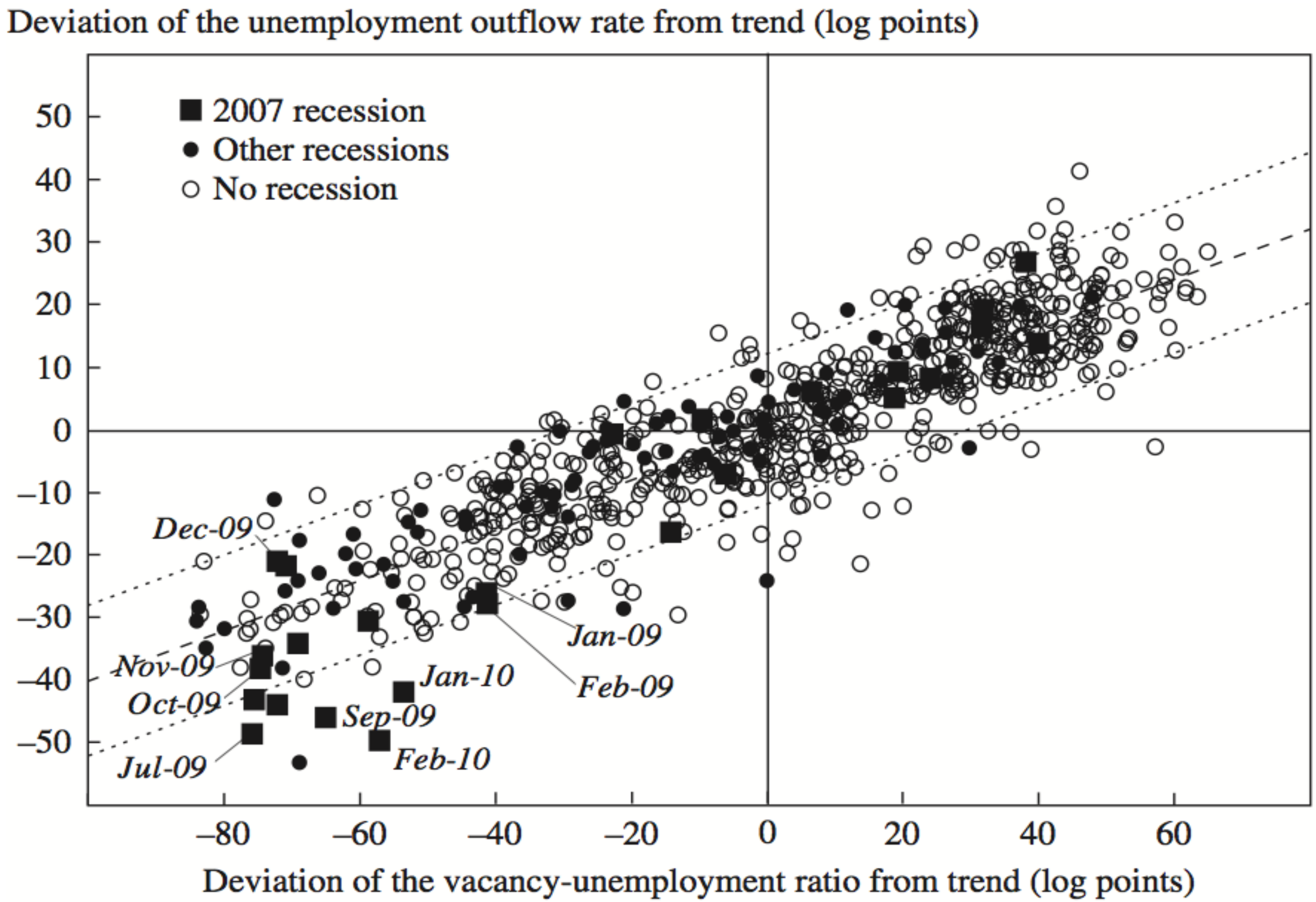
- Job finding rate $f(\theta)$, increasing in labor market tightness. Expected duration unemployment $1/f(\theta)$, decreasing in θ

- Vacancy filling rate $q(\theta)$, decreasing in labor market tightness. Expected duration vacancy $1/q(\theta)$, increasing in θ

- **Example**: if $F(u,v) = u^{\alpha}v^{1-\alpha}$ for $0 < \alpha < 1$ then

$$f(\theta) = \theta^{1-\alpha}, \qquad q(\theta) = \theta^{-\alpha}$$

Estimated matching function



Estimated relationship between job finding f and labor market tightness θ in postwar US data. Both expressed as log deviations from trend. A Cobb-Douglas matching function implies a *linear* relationship between $\log f$ and $\log \theta$.

Beveridge curve

- Now write steady state unemployment condition

$$\boxed{u = \frac{\delta}{\delta + f(\theta)}, \qquad \theta = v/u} \tag{1}$$

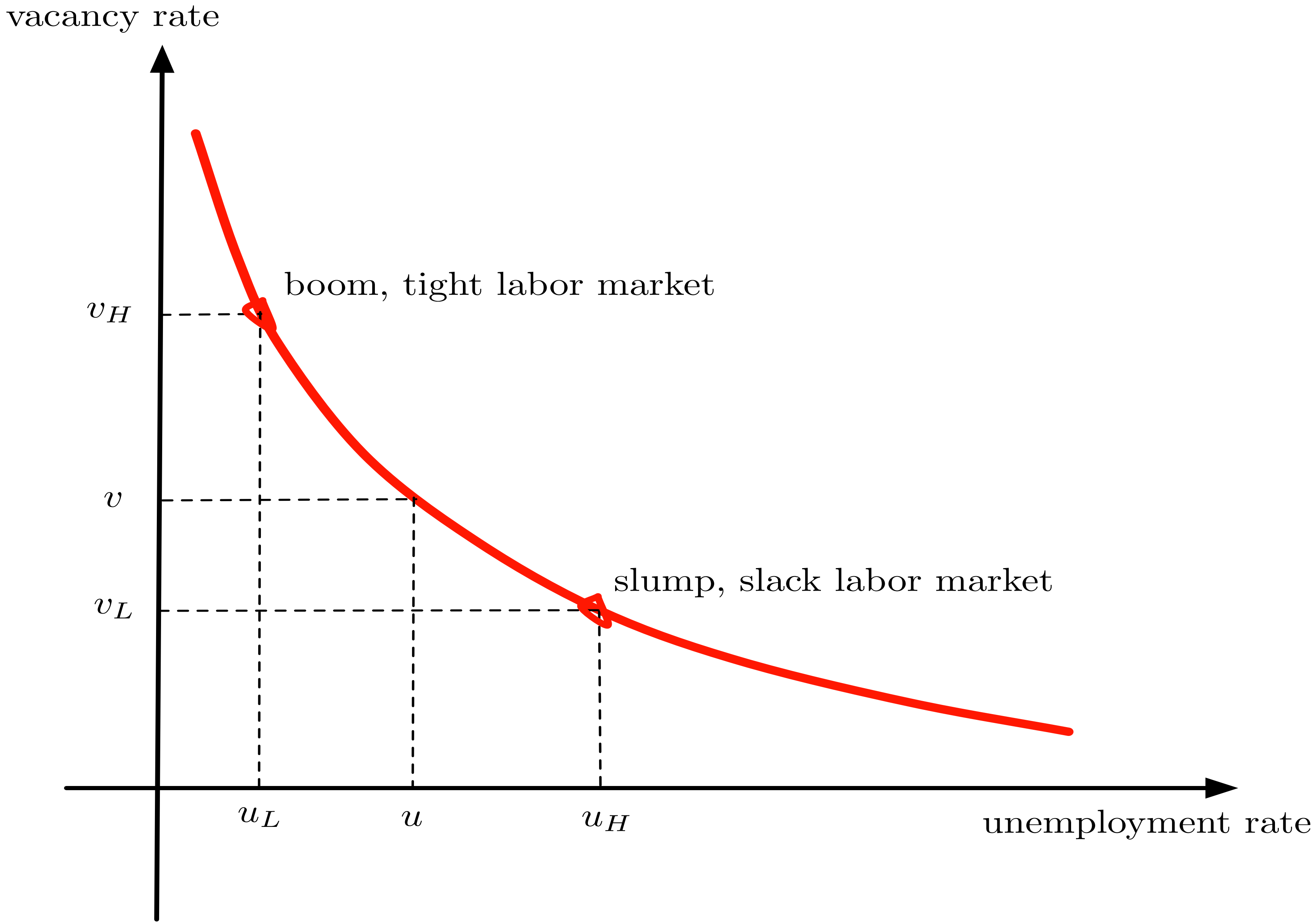
- Set of (v, u) satisfying (1) is known as the '*Beveridge curve*'

- An inverse relationship between v and u. Shifted by changes in the job destruction rate δ or the matching technology $f(\cdot)$

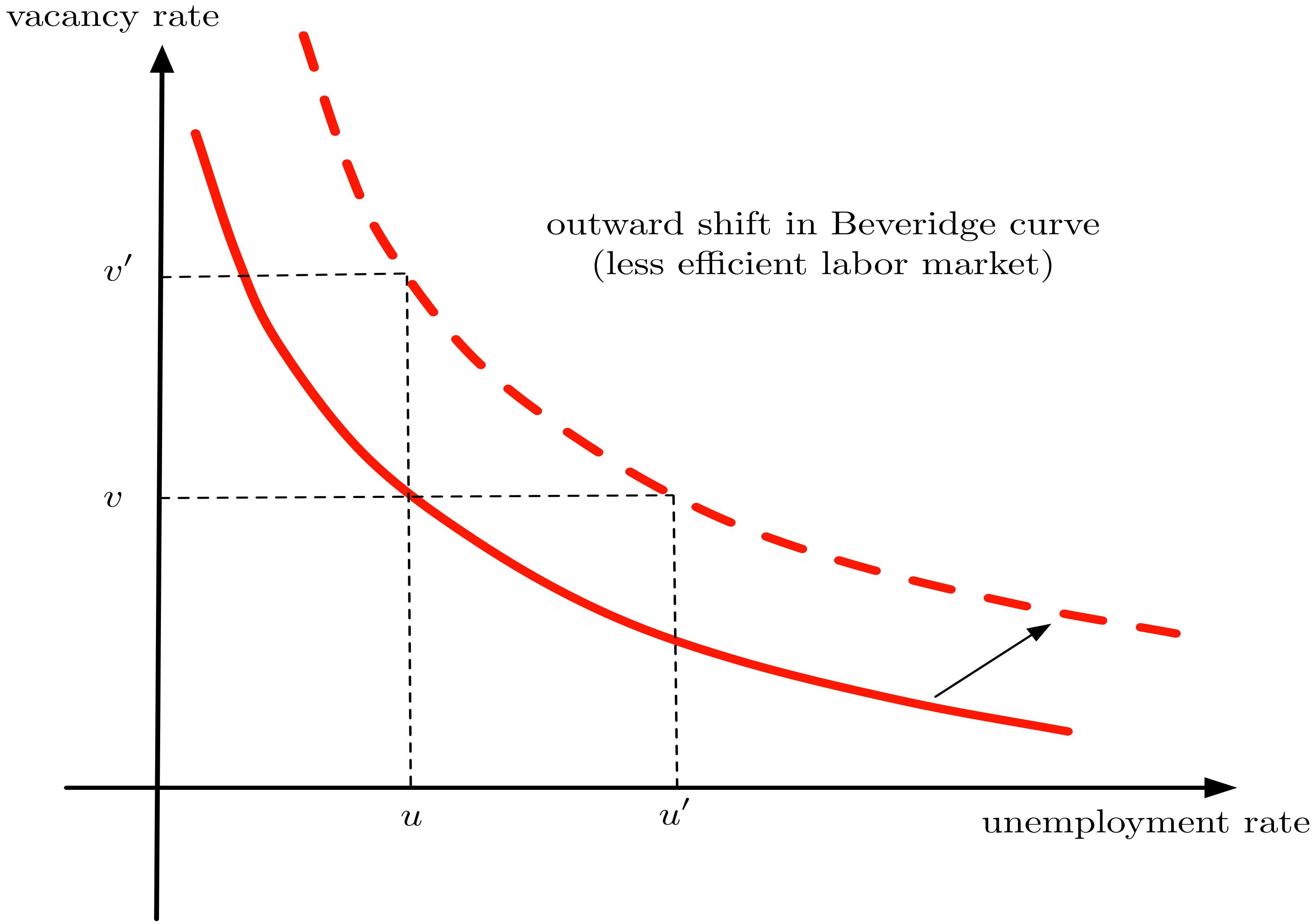
- **Example**: if $F(u, v) = a\, u^{\alpha} v^{1-\alpha}$ for $0 < \alpha < 1$ and $a > 0$ then

$$v = \left(\left(\frac{\delta}{a}\right)\left(\frac{1-u}{u^{\alpha}}\right)\right)^{1/(1-\alpha)}$$

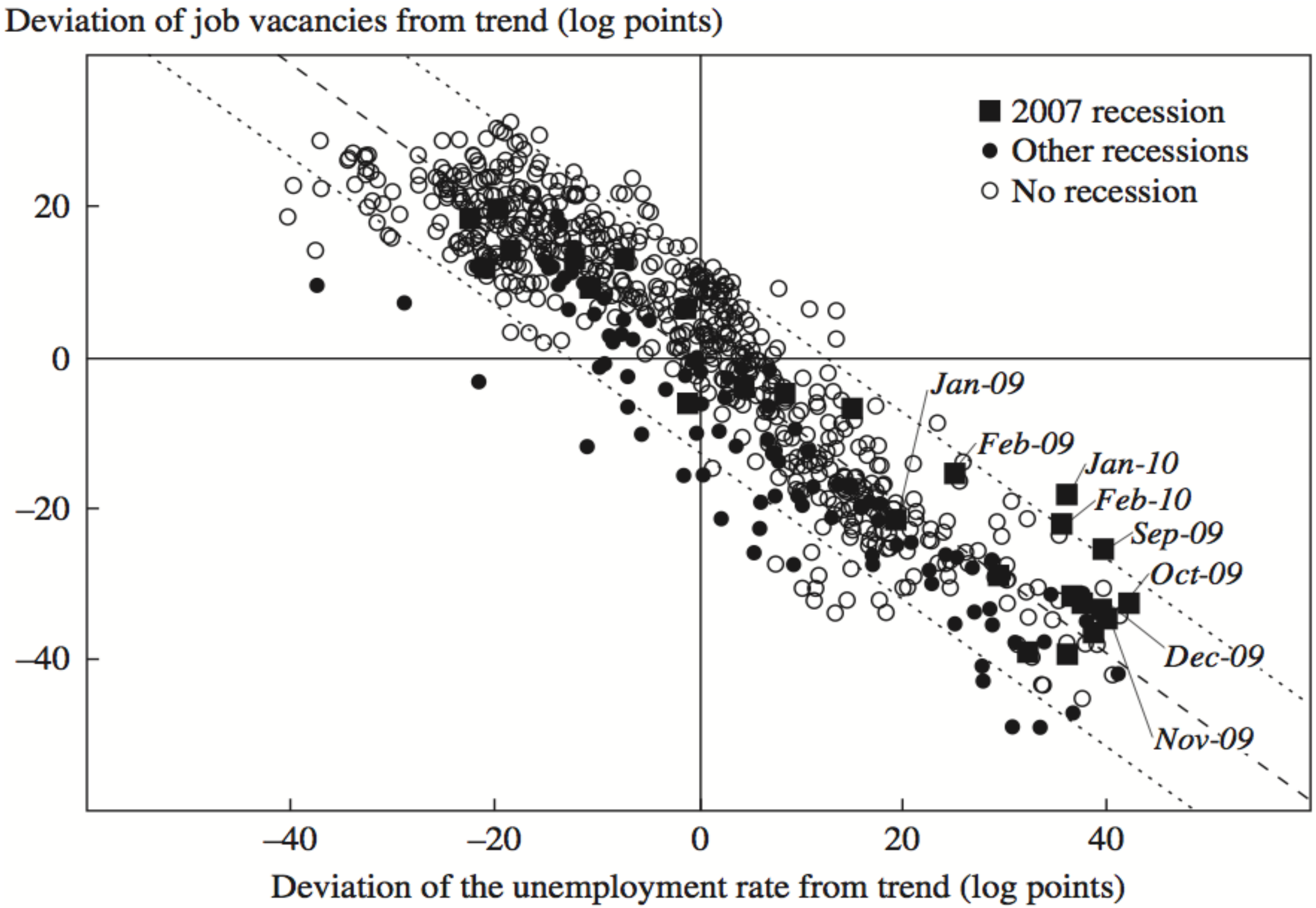
Beveridge curve



Shifts in the Beveridge curve



Estimated Beveridge curve



Estimated relationship between job vacancies v and unemployment u in postwar US data. Both expressed as log deviations from trend. This negative relationship is known as the Beveridge curve.

Pissarides (1985)

- Benchmark model of labor market flows
- Continuous time $t \geq 0$, will focus on steady states for now
- Risk neutral workers and firms, constant discount rate $r > 0$
- Unemployed workers and firms with vacancies matched via $F(u, v)$
- Free-entry into vacancy creation

Job creation and destruction

- Firms can employ one worker
- Flow value of production $z > 0$
- Flow wage paid to workers w
- Jobs destroyed at exogenous rate $\delta > 0$
- Jobs created by posting vacancies, flow cost $\kappa z > 0$
- Vacancy filling rate $q(\theta)$

Job creation

- Let J denote the present discounted value of a filled job. In steady state this satisfies the *Bellman equation*

$$rJ = z - w + \delta(V - J), \qquad \Rightarrow \qquad J = \frac{z - w}{r + \delta} + \frac{\delta}{r + \delta} V$$

- Let V denote the present discounted value of a vacancy

$$rV = -z\kappa + q(\theta)(J - V)$$

- Free-entry into job creation implies $V = 0$, so

$$J = \frac{z\kappa}{q(\theta)}$$

- Together these imply the 'marginal productivity' condition

$$\boxed{w = z - (r + \delta)\frac{z\kappa}{q(\theta)}} \tag{2}$$

- For given wage w, this will determine labor market tightness θ.

Workers

- Let W denote the present discounted value of being employed

$$rW = w + \delta(U - W), \qquad \Rightarrow \qquad W = \frac{w}{r+\delta} + \frac{\delta}{r+\delta}U$$

- Let U denote the present discounted value of being unemployed

$$rU = b + f(\theta)(W - U)$$

where $b \leq w$ denotes flow value of unemployment benefits etc

Wage determination

- Match between unemployed worker and firm with vacancy creates a mutual profit opportunity. How should these profits be split?
- Flow payments $z - w$ to firm, w to worker
- Wage w determined by *bargaining* between worker and firm
- Choice of w affects job value to individual firm $J(w)$ and to individual worker $W(w)$ taking as given aggregate market conditions U, V etc
- At a wage of w, the firm's surplus from a match is $J(w) - V$ and the worker's surplus is $W(w) - U$

Generalized Nash bargaining

- Wage w maximizes the *Nash product*

$$(W(w) - U)^{\beta}(J(w) - V)^{1-\beta}, \qquad 0 \leq \beta \leq 1$$

where the parameter β denotes the workers' *bargaining power*

- First order condition for this problem can be written

$$\beta \frac{W'(w)}{W(w) - U} = -(1 - \beta) \frac{J'(w)}{J(w) - V}$$

Now note that, treating aggregate U, V as given

$$W'(w) = \frac{1}{r + \delta}, \qquad J'(w) = -\frac{1}{r + \delta} = -W'(w)$$

- So we can write

$$W = U + \beta S$$

where $S = W - U + J$ is the total match surplus (given $V = 0$)

Wages and the value of unemployment

- Recall that

$$W = \frac{w + \delta U}{r + \delta}, \qquad \text{and} \qquad J = \frac{z - w}{r + \delta}$$

- Then given surplus splitting $W - U = \beta(W - U + J)$ we have

$$\frac{w}{r+\delta} - \frac{r}{r+\delta}U = \beta\left(\frac{w}{r+\delta} - \frac{r}{r+\delta}U + \frac{z-w}{r+\delta}\right)$$

- Hence in current value terms

$$w - rU = \beta(w - rU + z - w)$$

which implies

$$w = rU + \beta(z - rU)$$

Wage curve

- Hence

$$w = (1-\beta)rU + \beta z$$

- Which by the Bellman equation for U is also

$$w = (1-\beta)[b + f(\theta)(W-U)] + \beta z$$

- But worker surplus proportional to firm surplus which is pinned down by free entry

$$W - U = \frac{\beta}{1-\beta}J = \frac{\beta}{1-\beta}\frac{z\kappa}{q(\theta)}$$

- With a bit more algebra and using $f(\theta)/q(\theta) = \theta$ we get

$$\boxed{w = (1-\beta)b + \beta(1+\kappa\theta)z} \qquad (3)$$

(which is known as the '*wage curve*')

Steady state equilibrium

- Steady state (u, w, θ) solves

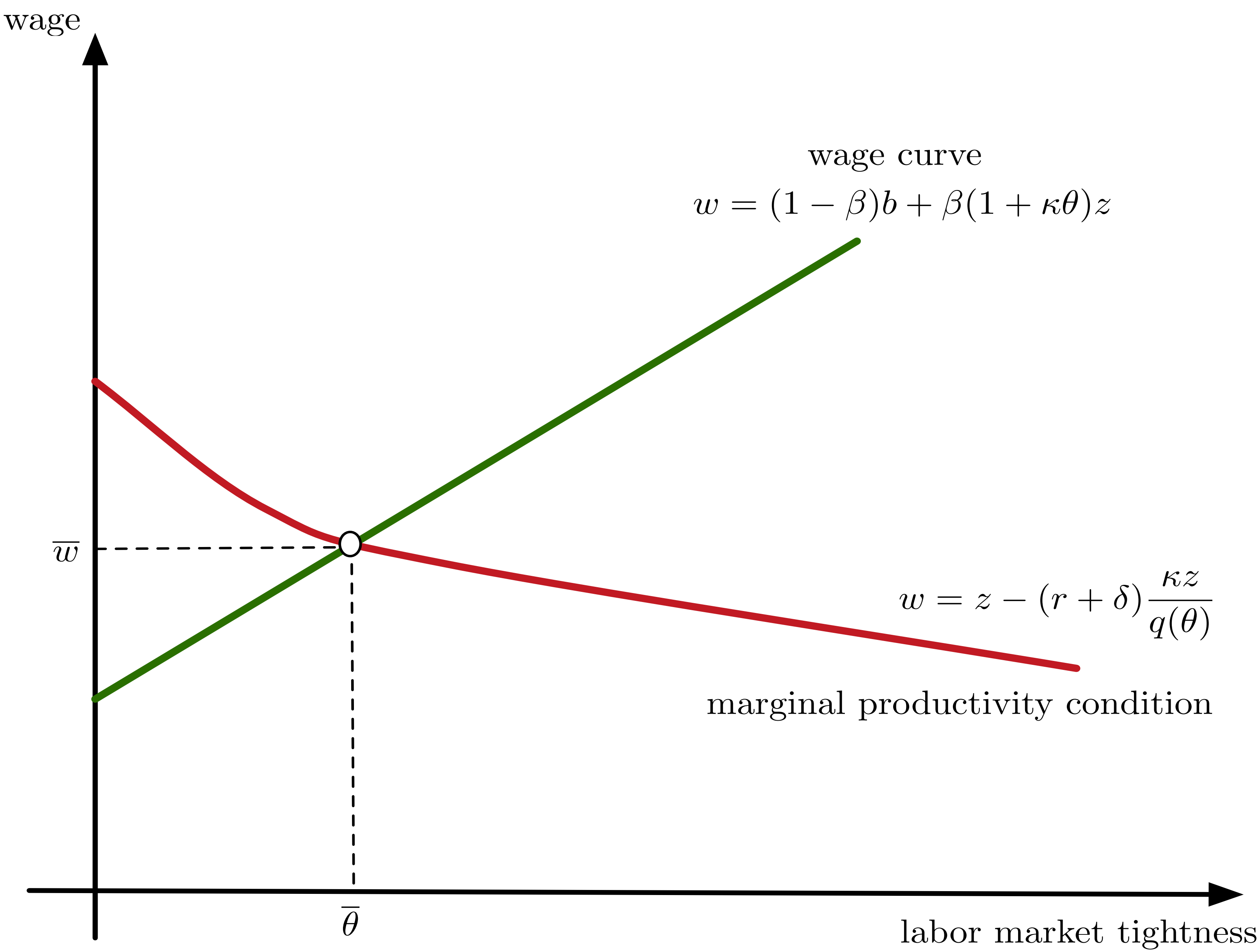
$$(1) \qquad w = (1-\beta)b + \beta(1+\kappa\theta)z \qquad \text{(wage curve)}$$

$$(2) \qquad w = z - (r+\delta)\frac{\kappa z}{q(\theta)} \qquad \text{(marginal productivity)}$$

$$(3) \qquad u = \frac{\delta}{\delta + f(\theta)} \qquad \text{(Beveridge curve)}$$

- Solve (1) and (2) simultaneously for w, θ. Then recover u from (3) and then $v = \theta u$ etc. Recover W, U, J from Bellman equations.

Steady state w, θ



Recover u from Beveridge curve