Advanced Macroeconomics

Lecture 18: monetary economics, part six

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This class

- Optimal monetary policy in a liquidity trap *with commitment*
- Further reading
    - Werning (2012): Managing a liquidity trap: Monetary and fiscal policy, MIT working paper, section 4

Optimal monetary policy with commitment

- Monetary policy minimizes

$$L = \frac{1}{2}\int_0^{\infty} e^{-\rho t}\left(x(t)^2 + \lambda \pi(t)^2\right) dt$$

subject to the constraints

$$\begin{aligned}
\dot{x}(t) &= \sigma^{-1}(i(t) - \pi(t) - r^n(t)) \\
\dot{\pi}(t) &= \rho\pi(t) - \kappa x(t) \\
i(t) &\geq 0
\end{aligned}$$

taking as given path of natural real rate $r^n(t)$

- Control i, state x, π with free initial conditions $x(0), \pi(0)$

Optimal monetary policy with commitment

- Hamiltonian for this problem

$$\mathcal{H} = \frac{1}{2}(x^2 + \lambda\pi^2) + \mu_x(\sigma^{-1}(i - \pi - r^n)) + \mu_\pi(\rho\pi - \kappa x) - \psi i$$

with multipliers μ_x, μ_π and multiplier on ZLB constraint ψ

- Key optimality conditions

$$\mu_x(t)\sigma^{-1} = \psi(t), \qquad \psi(t)i(t) = 0 \qquad \text{with comp. slackness}$$

and

$$\begin{aligned}
\rho\mu_x(t) - \dot{\mu}_x(t) &= x(t) - \kappa\mu_\pi(t) \\
\rho\mu_\pi(t) - \dot{\mu}_\pi(t) &= \lambda\pi(t) - \sigma^{-1}\mu_x(t) + \rho\mu_\pi(t)
\end{aligned}$$

Optimal monetary policy with commitment

- Hence system can be written

$$\mu_x(t) \geq 0, \qquad \mu_x(t)i(t) = 0$$

with

$$\begin{aligned}
\dot{\mu}_x(t) &= \rho\mu_x(t) - x(t) + \kappa\mu_\pi(t) \\
\dot{\mu}_\pi(t) &= -\lambda\pi(t) + \sigma^{-1}\mu_x(t) \\
\dot{x}(t) &= \sigma^{-1}(i(t) - \pi(t) - r^n(t)) \\
\dot{\pi}(t) &= \rho\pi(t) - \kappa x(t)
\end{aligned}$$

taking as given path natural real rate $r^n(t)$

- Boundary conditions: (i) $\mu_x(0) = 0$ and $\mu_\pi(0) = 0$, since both $x(0)$ and $\pi(0)$ are free, and (ii) two transversality conditions

Preliminaries

- Suppose ZLB not binding, $\psi(t) = 0$ hence $\mu_x(t) = \dot{\mu}_x(t) = 0$ so that

$$x(t) = \kappa\mu_\pi(t)$$

- Implies familiar targeting rule, as in Lecture 16 slide 15

$$\dot{x}(t) = \kappa\dot{\mu}_\pi(t) = \kappa\Big(-\lambda\pi(t) + \sigma^{-1}0\Big) = -\kappa\lambda\pi(t)$$

- But from the dynamic IS curve

$$\dot{x}(t) = \sigma^{-1}(i(t) - \pi(t) - r(t))$$

- Solving for $i(t)$ then gives

$$i(t) = I(r^n(t), \pi(t)), \quad \text{where} \quad I(r, \pi) := r + (1 - \sigma\kappa\lambda)\pi$$

This is the optimal nominal rate *whenever the ZLB is not binding.* $I(r, \pi) \geq 0$ is necessary for ZLB to not bind. But not sufficient.

Approach

- Three phases

  I. During the liquidity trap, $t \in [0, T)$

  II. Just out of the trap, $t \in [T, \hat{T})$, some endogenous $\hat{T} \geq T$

  III. After the storm has passed, $t \in [\hat{T}, \infty)$

- Need to 'stitch together' three phase diagrams

- Key is whether $x(t), \pi(t)$ are free at critical dates $t = 0, T, \hat{T}$

- Solve backwards from terminal conditions

Phase III. After the storm

- At beginning of Phase III $x(\hat{T}), \pi(\hat{T})$ are given (not free)
- ZLB is *not binding* so $i(t) = I(\bar{r}, \pi(t))$
- Under this control, motion of system given by

$$\dot{x}(t) = -\kappa\lambda\pi(t)$$
$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

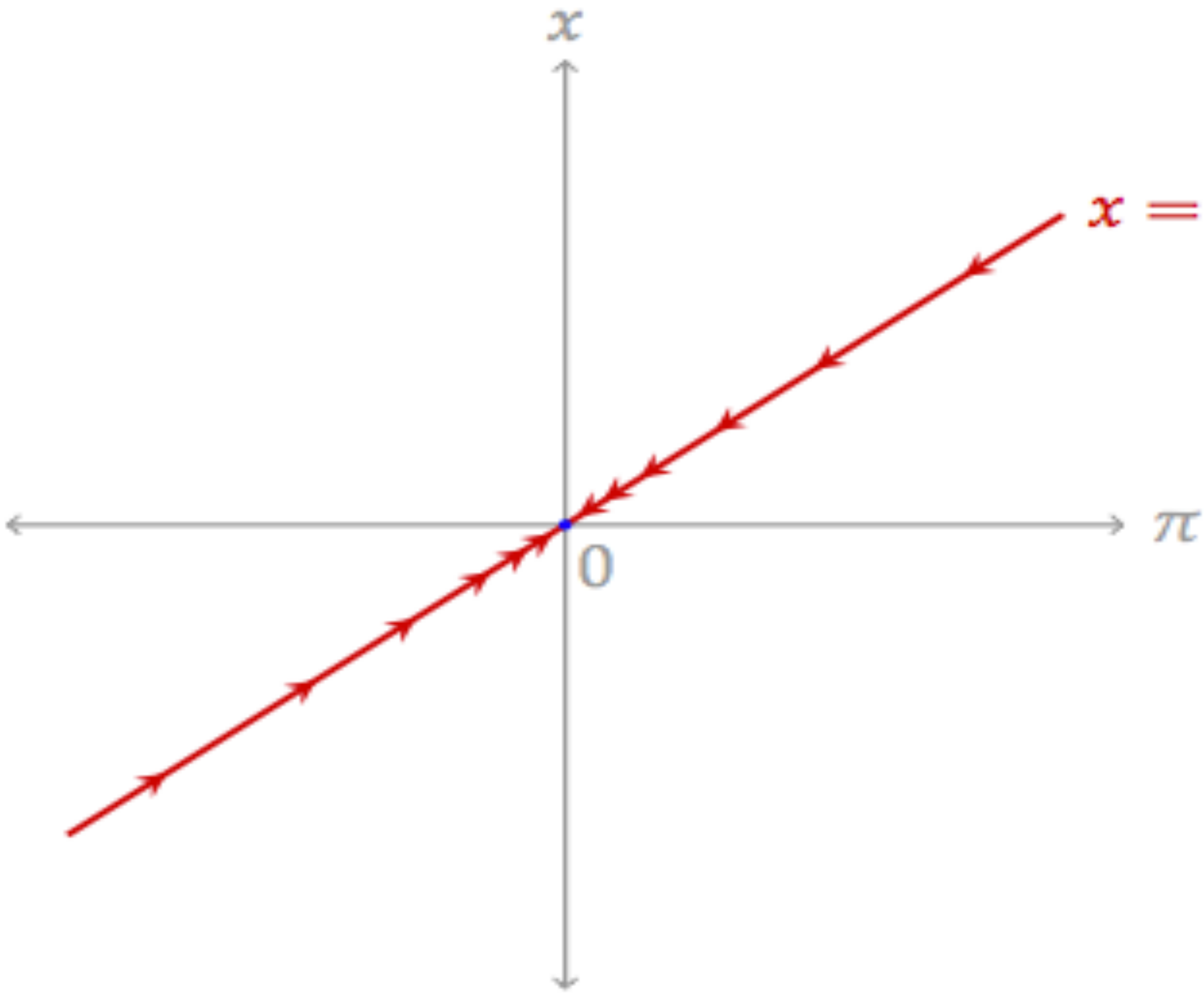
- Solve with method of undetermined coefficients

$$x(t) = \phi\pi(t)$$

for some ϕ to be determined, see appendix for details

Phase III. After the storm

ϕ is slope of saddle-path through $(0,0)$ with $i(t) = I(\bar{r}, \pi(t))$ for $t \in [\hat{T}, \infty)$



Phase II. Just out of the trap

- At beginning of Phase II $x(T), \pi(T)$ are given (not free)

- Liquidity trap is over but $i(t) = 0$ is *still optimal.* Policy commits to keeping $i(t) = 0$ even after trap is over

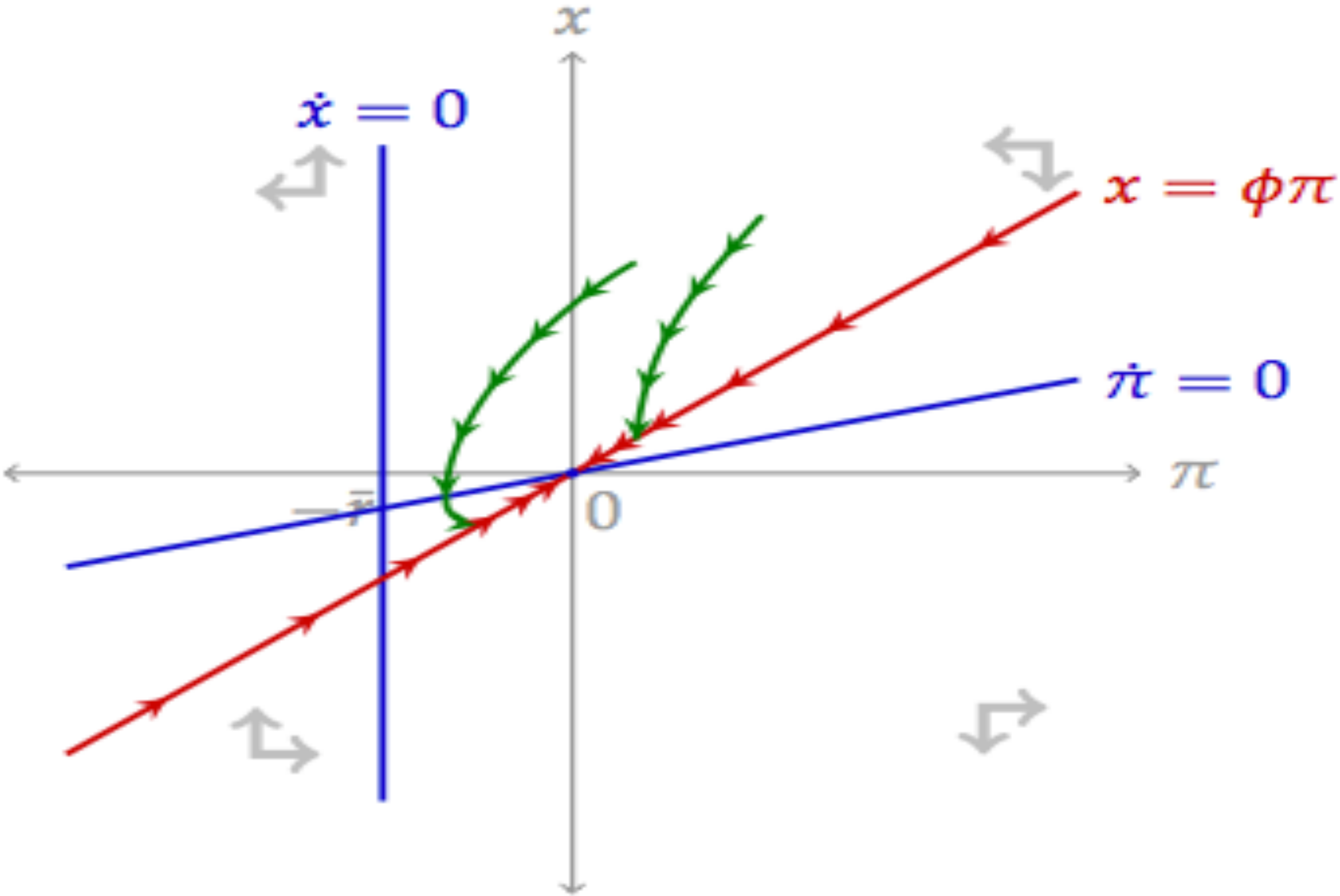
- Motion of system given by

$$\dot{x}(t) = -\sigma^{-1}(\pi(t) + \overline{r})$$
$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

- Same phase diagram as discretionary case, except $\dot{x}(t) = 0$ locus at $\pi(t) = -\overline{r} < 0$ rather than at $-\underline{r} > 0$

Phase II. Just out of the trap

Admissible dynamics *from* given $x(T), \pi(T)$ with $i(t) = 0$ for $t \in [T, \hat{T})$



Phase I. During the liquidity trap

- At beginning of Phase I $x(0), \pi(0)$ free, but $x(T), \pi(T)$ given

- ZLB is *binding*, $i(t) = 0$

- Motion of system given by

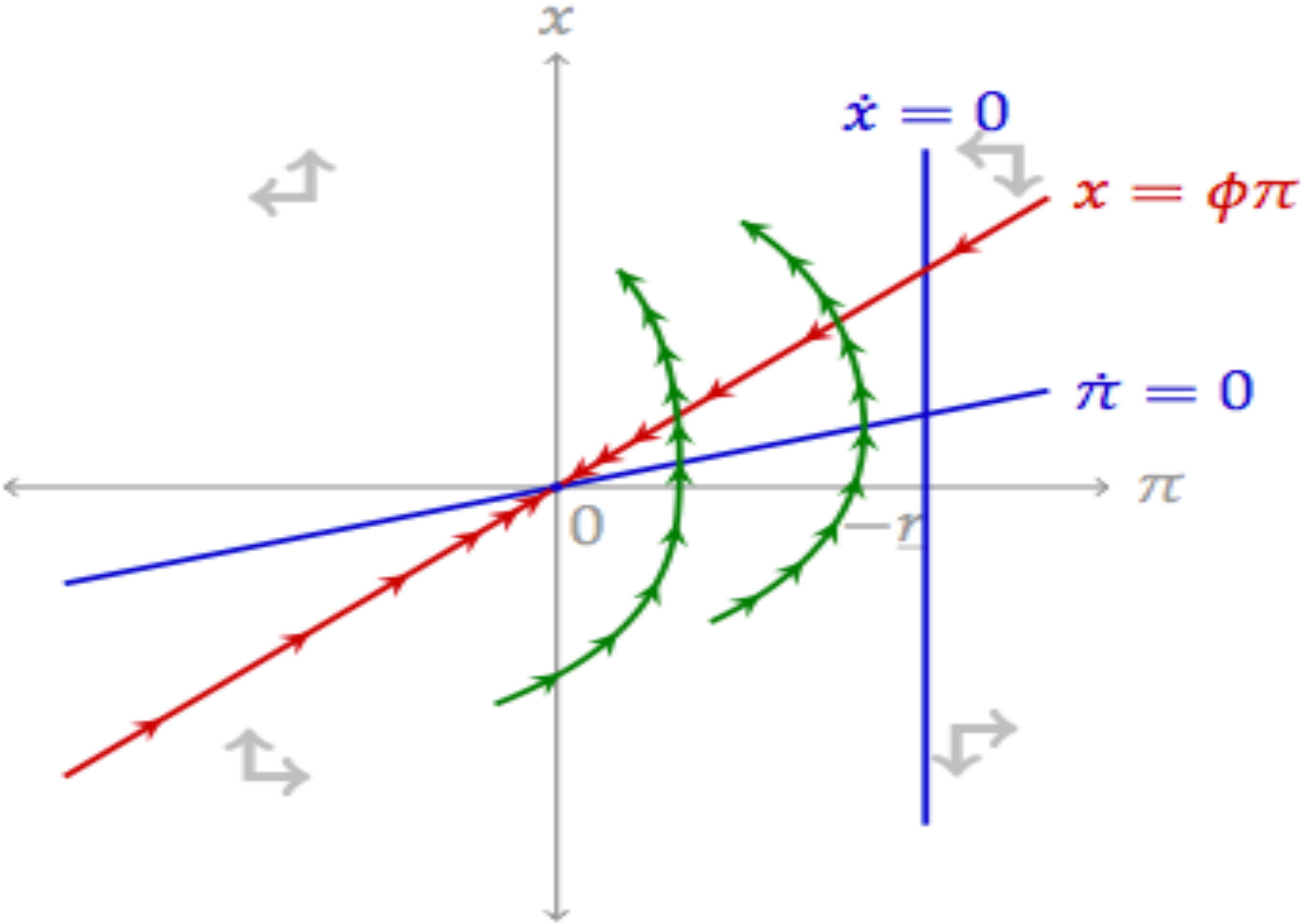
$$\dot{x}(t) = -\sigma^{-1}(\pi(t) + \underline{r})$$
$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

- Exact same phase diagram as discretionary case, $\dot{x}(t) = 0$ locus at $\pi(t) = -\underline{r} > 0$ etc. *But different terminal conditions*

  – with discretion $x(T), \pi(T) = (0, 0)$
  – with commitment $x(T), \pi(T)$ given from Phase II.

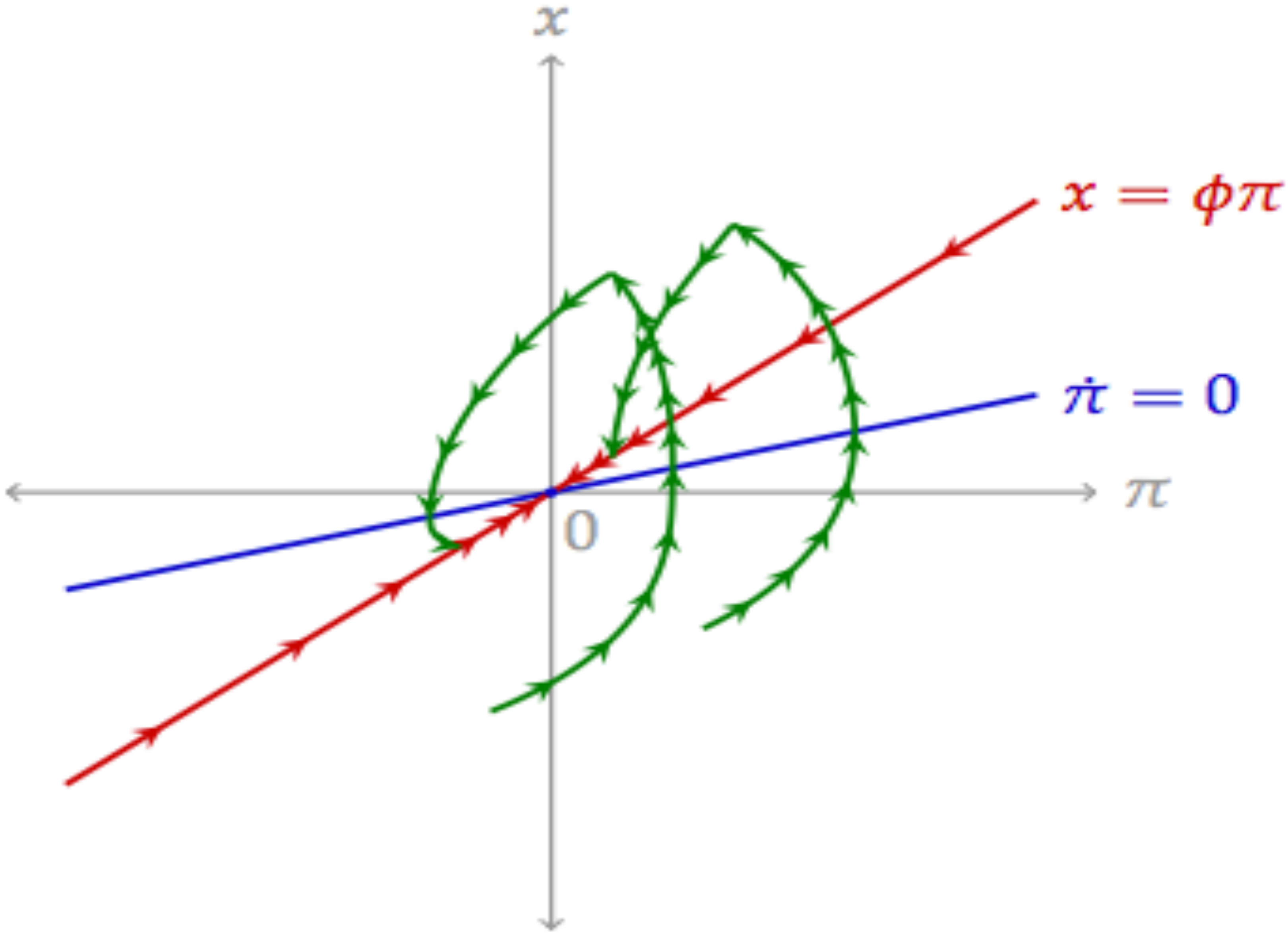
Phase I. During the liquidity trap

Admissible dynamics *towards* $x(T), \pi(T)$ with $i(t) = 0$ for $t \in [0, T)$



Stitching it all together

Dynamics through the entire episode



Summary

(1) If ZLB is not binding, then $i(t) = I(r^n(t), \pi(t))$

(2) If $I(r^n(t), \pi(t)) < 0$ for $t \in [0, T)$ then $i(t) = 0$ for $t \in [0, \hat{T})$ for some $\hat{T} \geq T$

(3) Inflation must be positive at some point

(4) Output must be both positive and negative

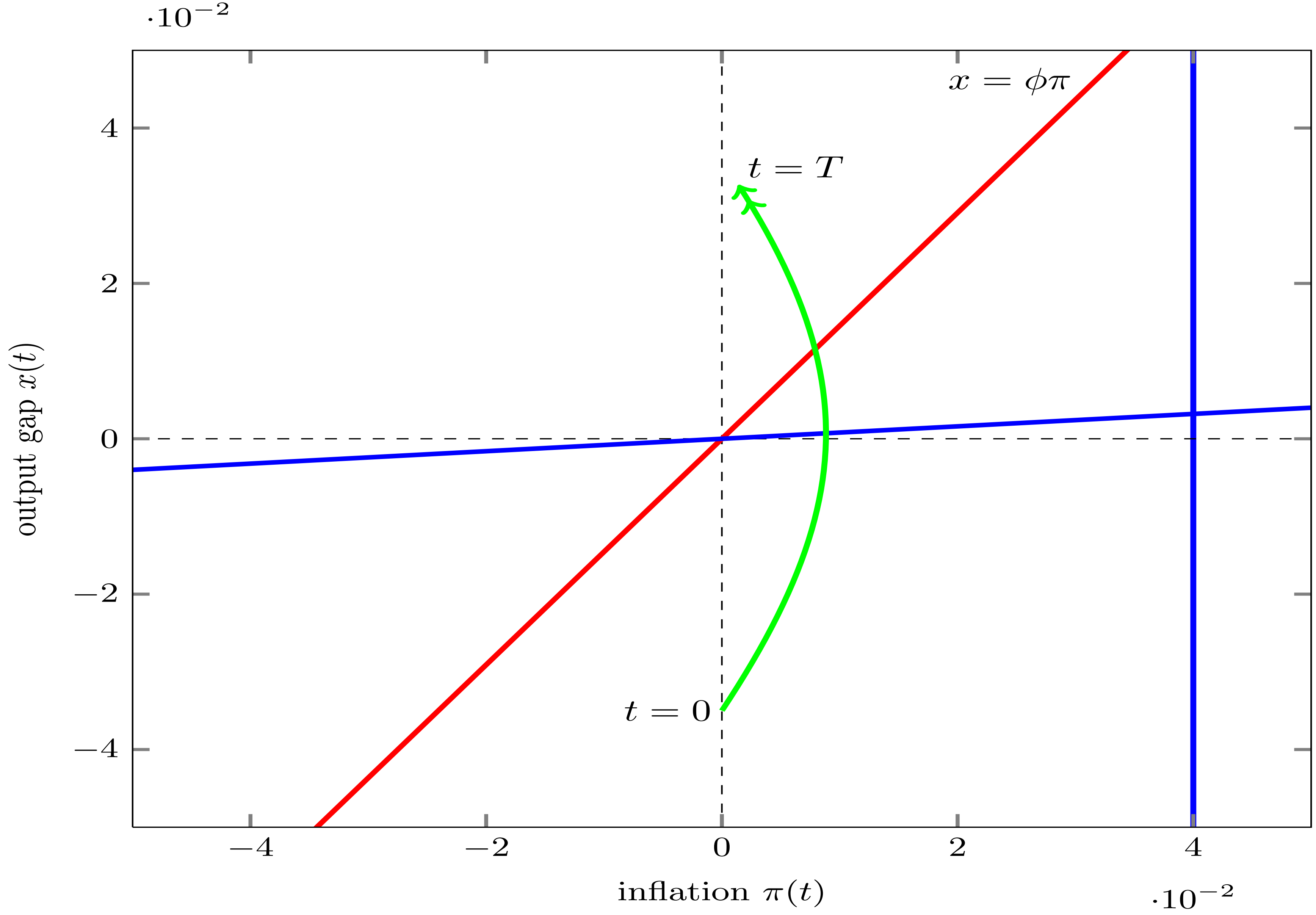
(5) Depending on parameters, inflation may be positive throughout

Numerical example

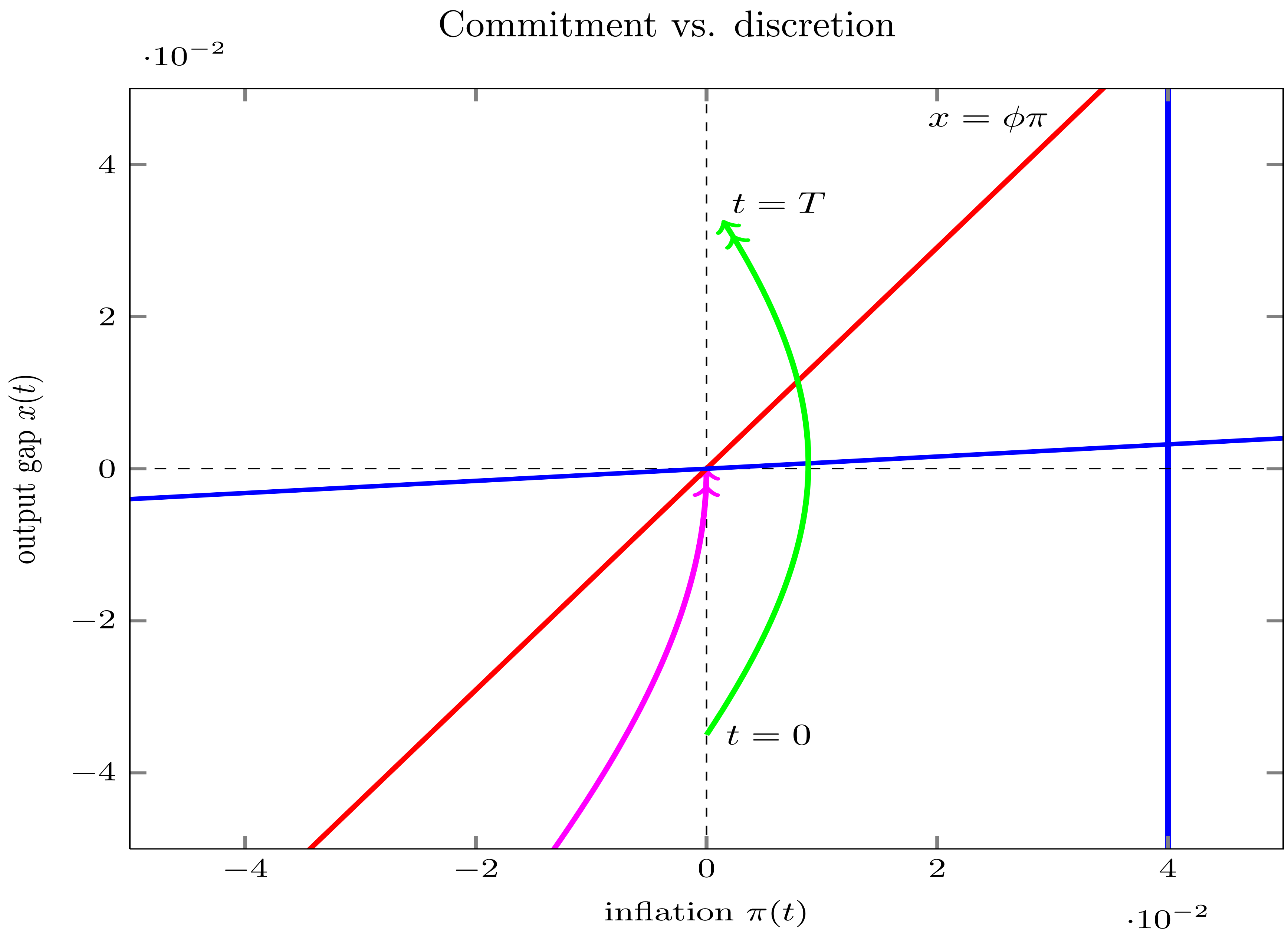
- Suppose $\sigma = 1$, $\kappa = 0.5$ and $\lambda = 1/\kappa = 2$
- This choice of parameters implies $\sigma\kappa\lambda = 1$ which implies $\pi(0) = 0$
- Obtain trajectory $x(t), \pi(t)$ for some given $x(0)$
- Optimize over $x(0)$ to find trajectory that minimizes loss

During the liquidity trap $t \in [0, T)$

$r^n(t) = \underline{r} < 0$, ZLB binding $i(t) = 0$

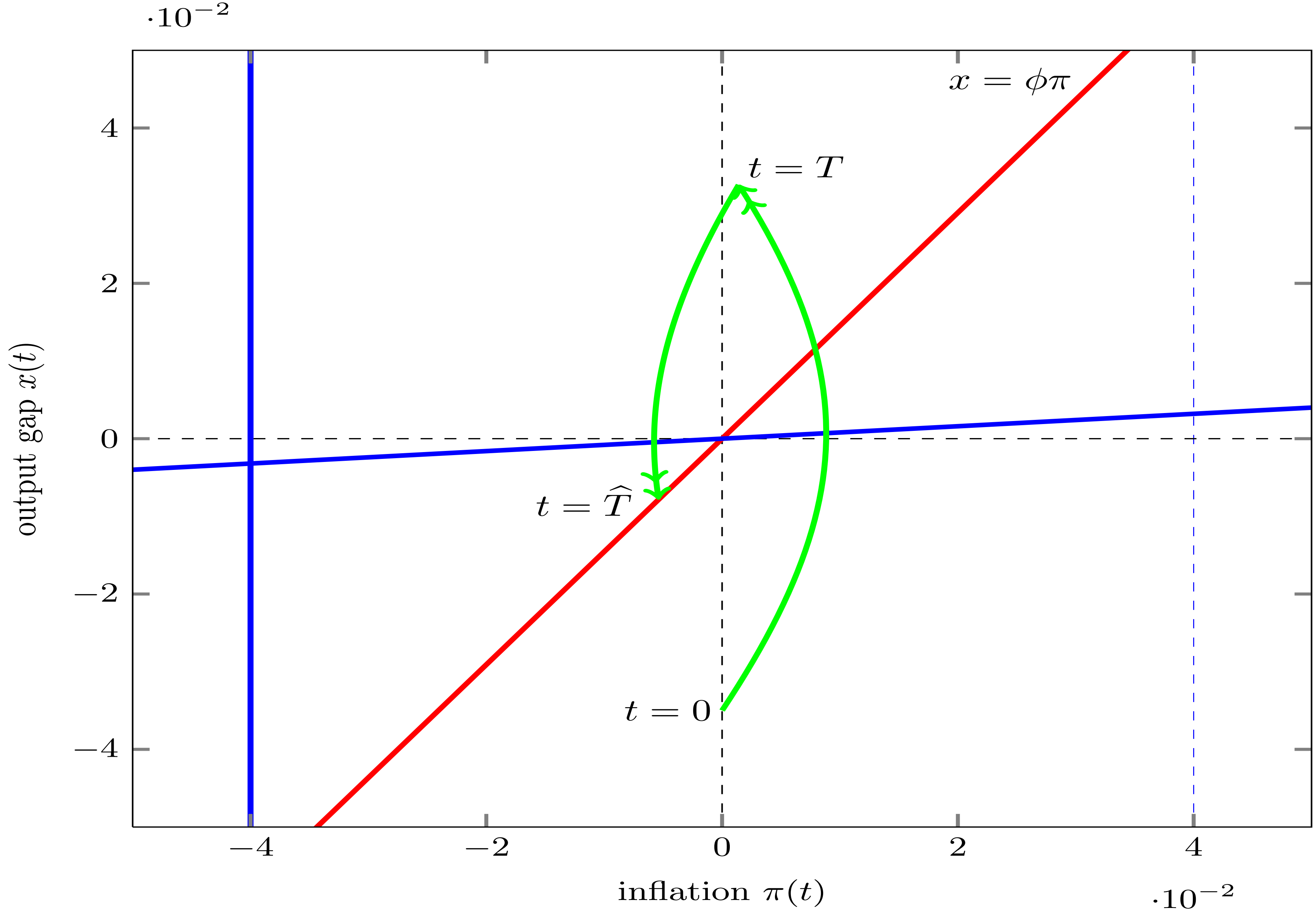


During the liquidity trap $t \in [0, T)$



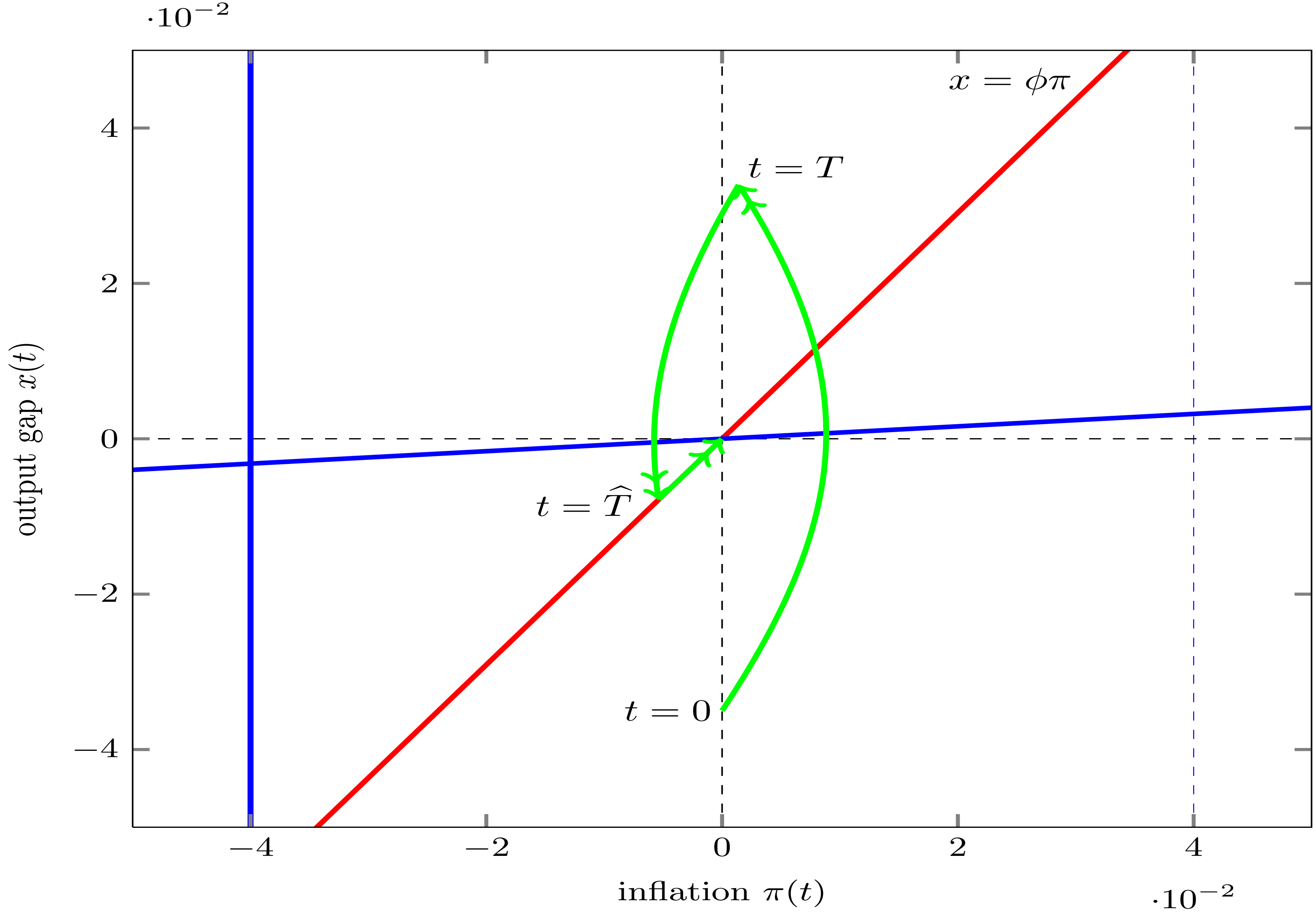
Just out of the liquidity trap $t \in [T, \hat{T})$

$r^n(t)$ back to $\overline{r} > 0$ but still $i(t) = 0$

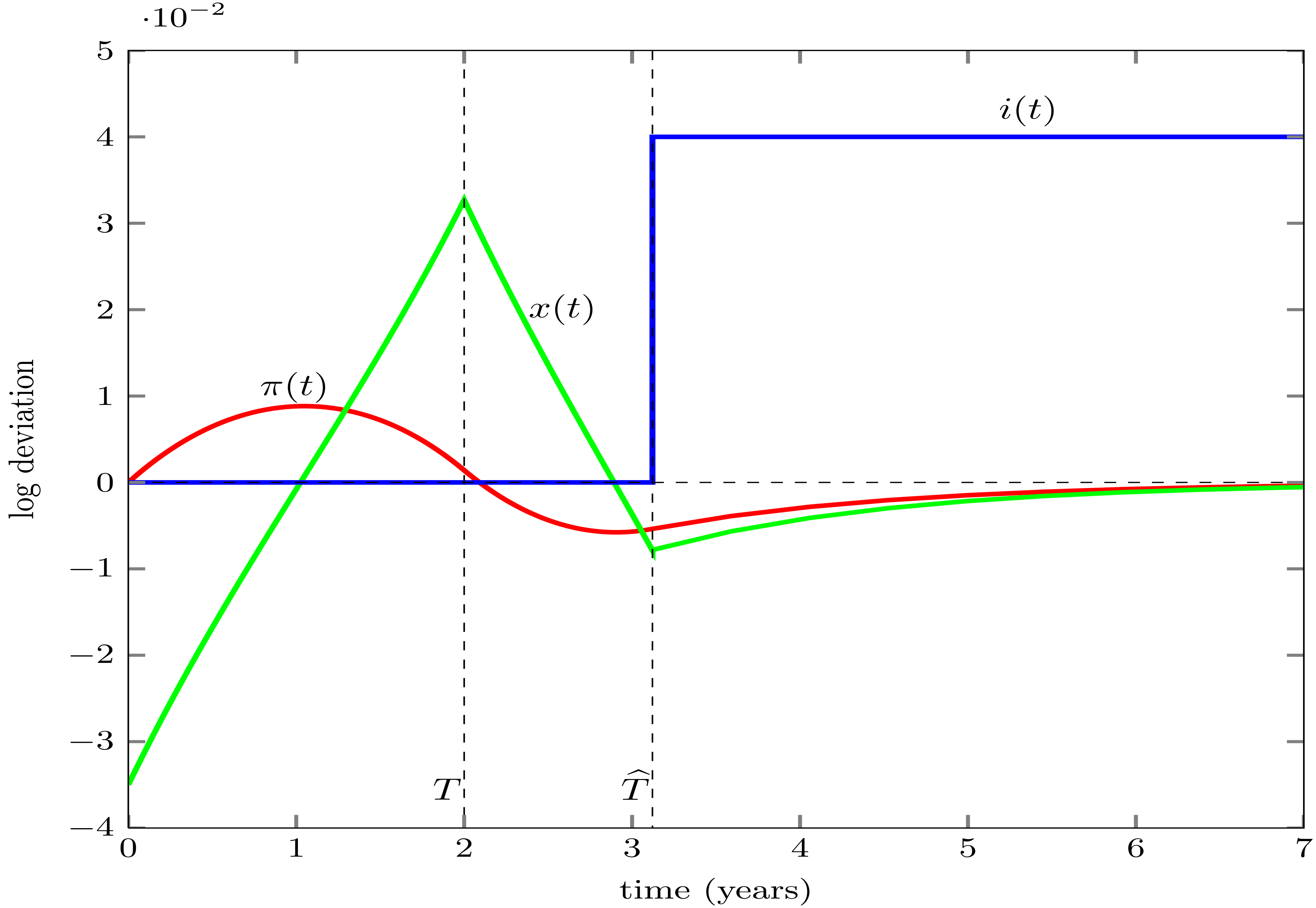


After the storm has passed $t \in [\hat{T}, \infty)$

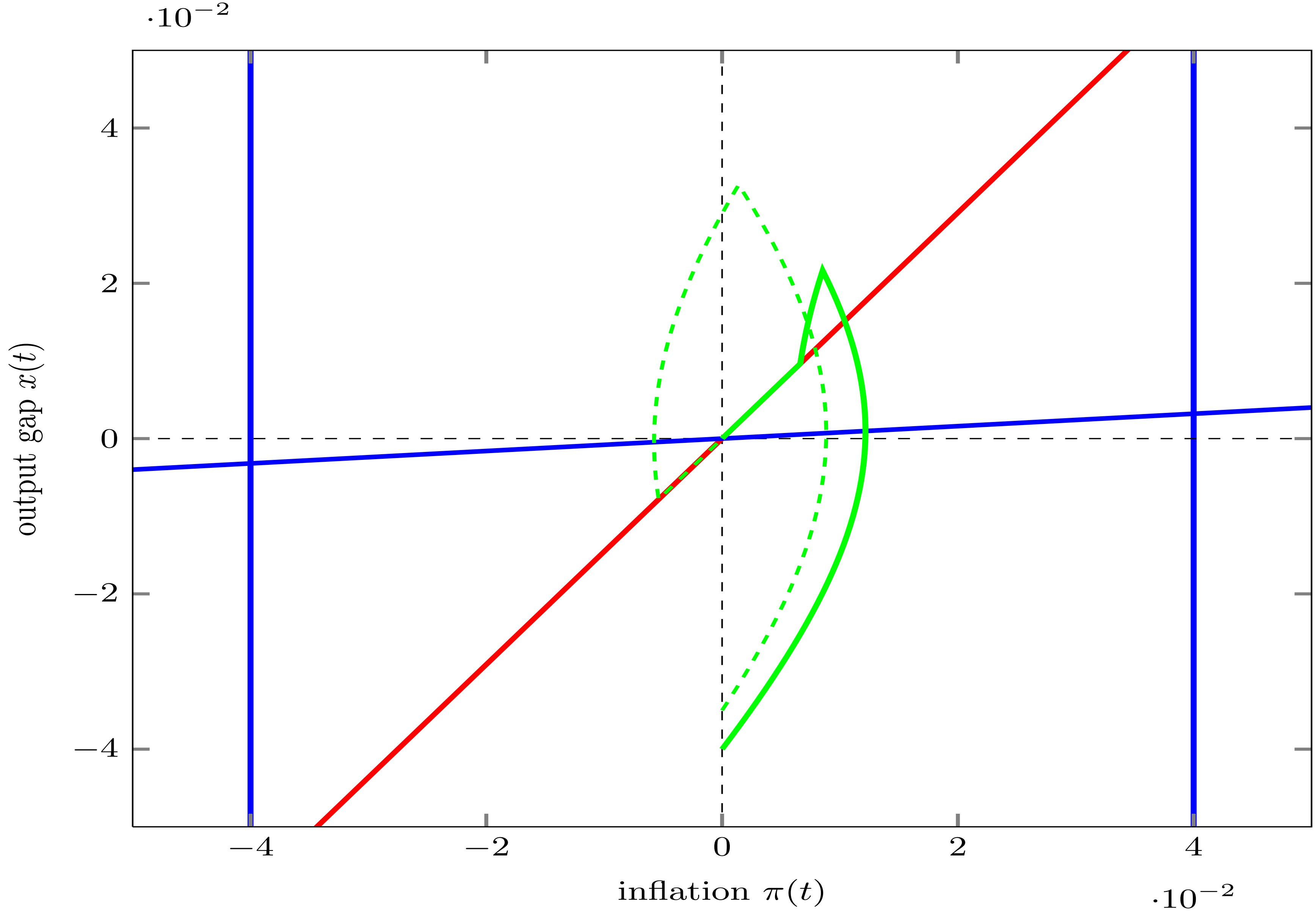
$r^n(t)$ at $\overline{r} > 0$ and now $i(t) = I(\overline{r}, \pi(t))$



Nominal rate $i(t)$ jumps at $\hat{T}$, not T



For each initial $x(0)$, a different trajectory



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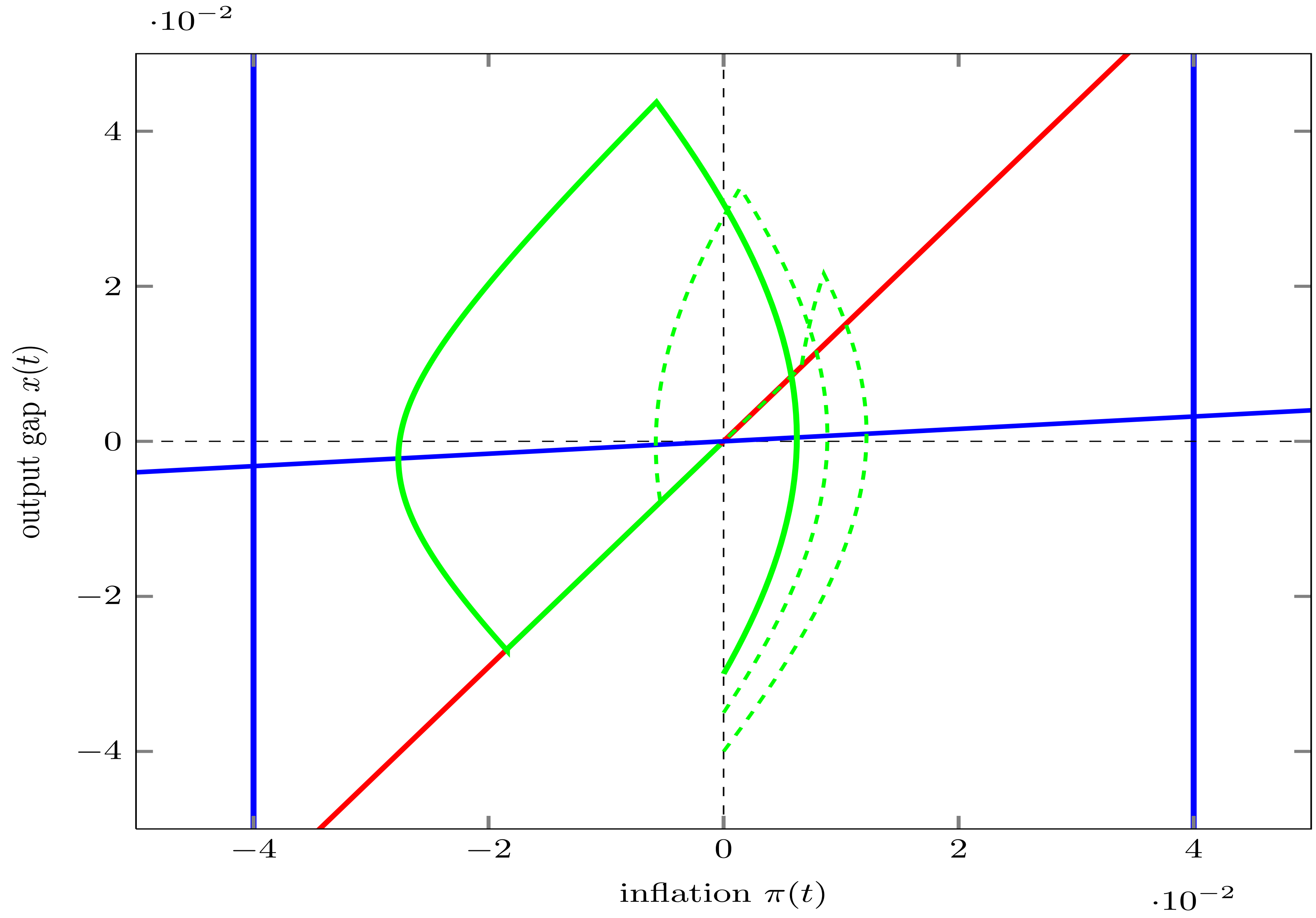
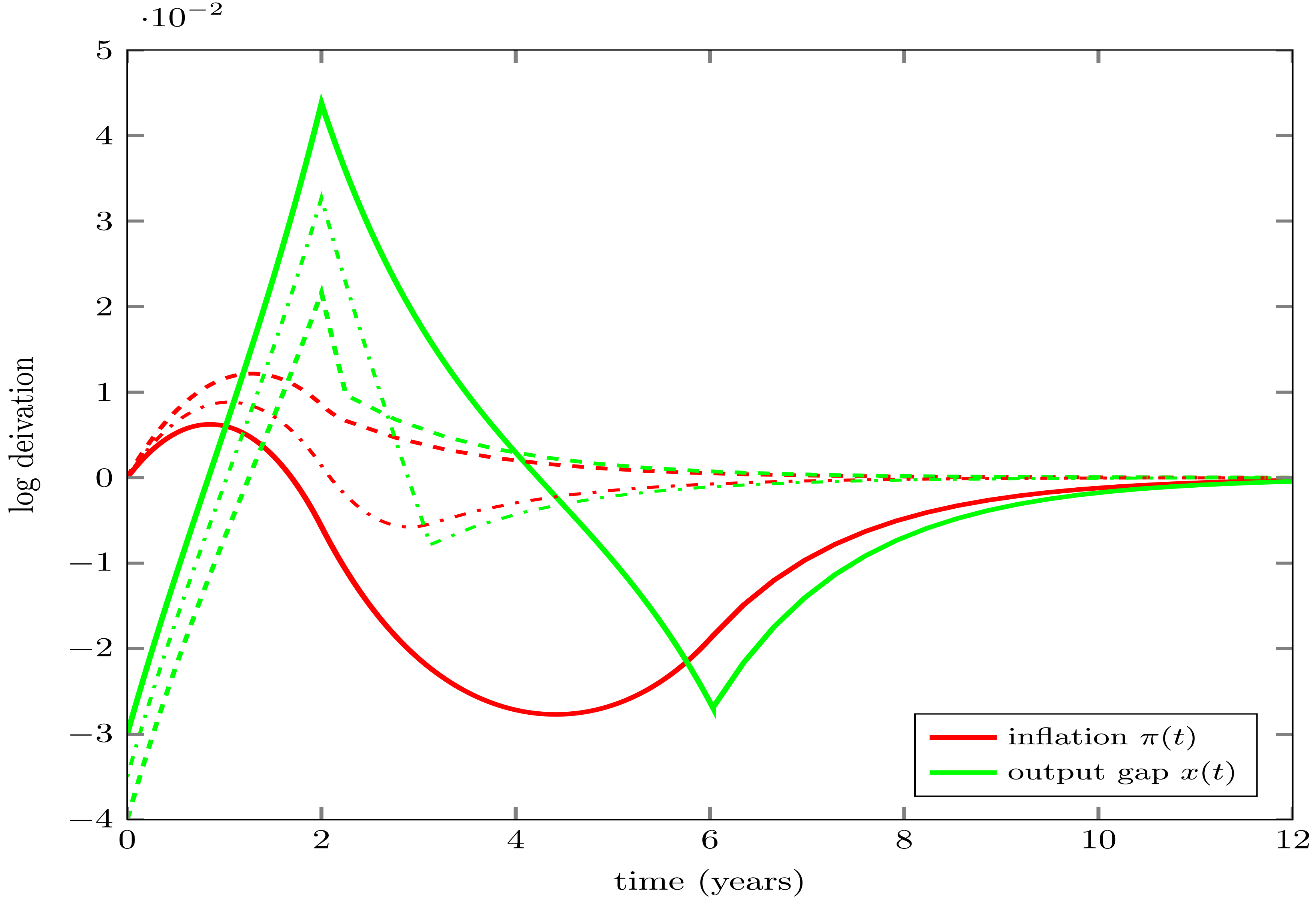
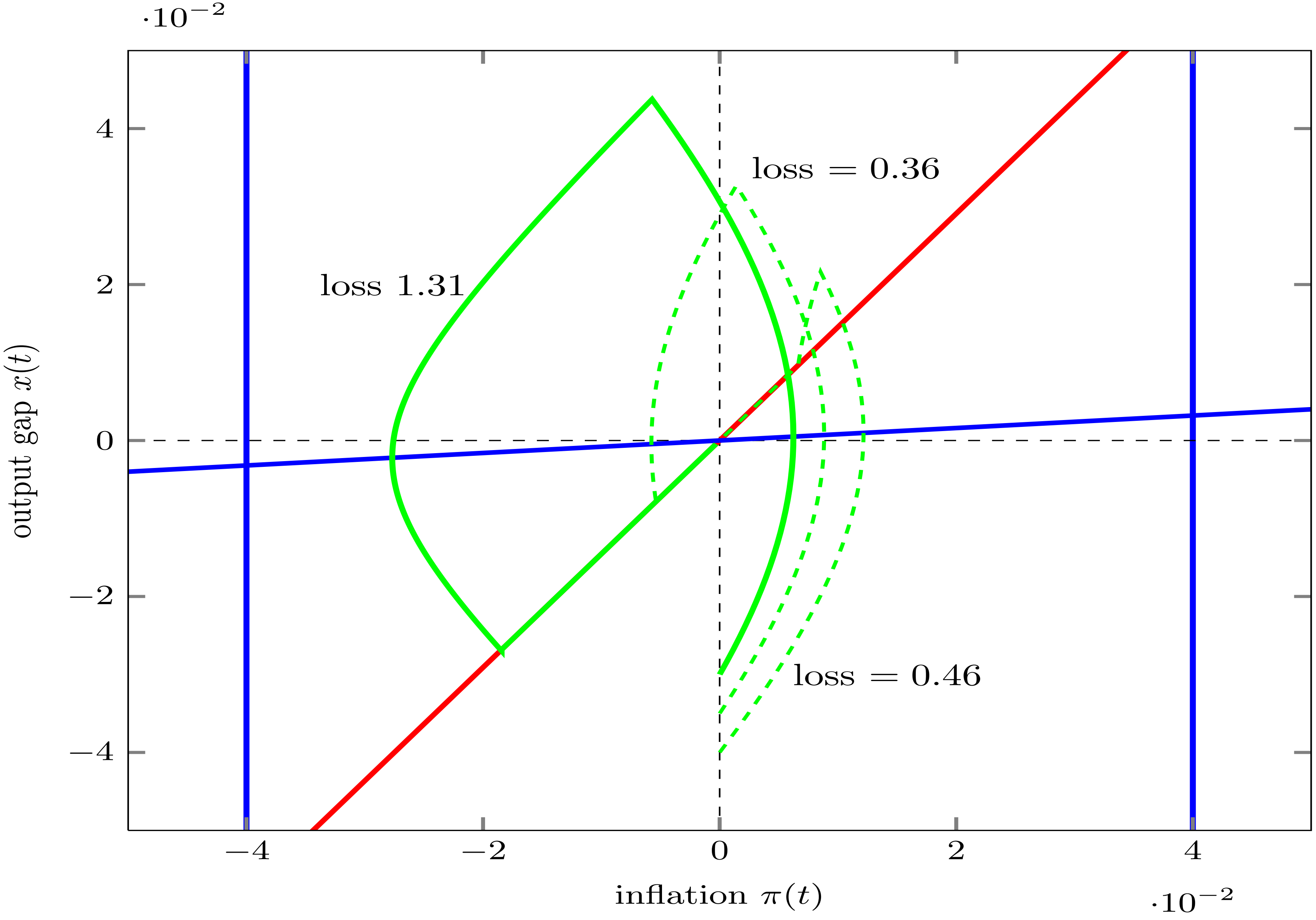


Tradeoff between initial recession and volatility

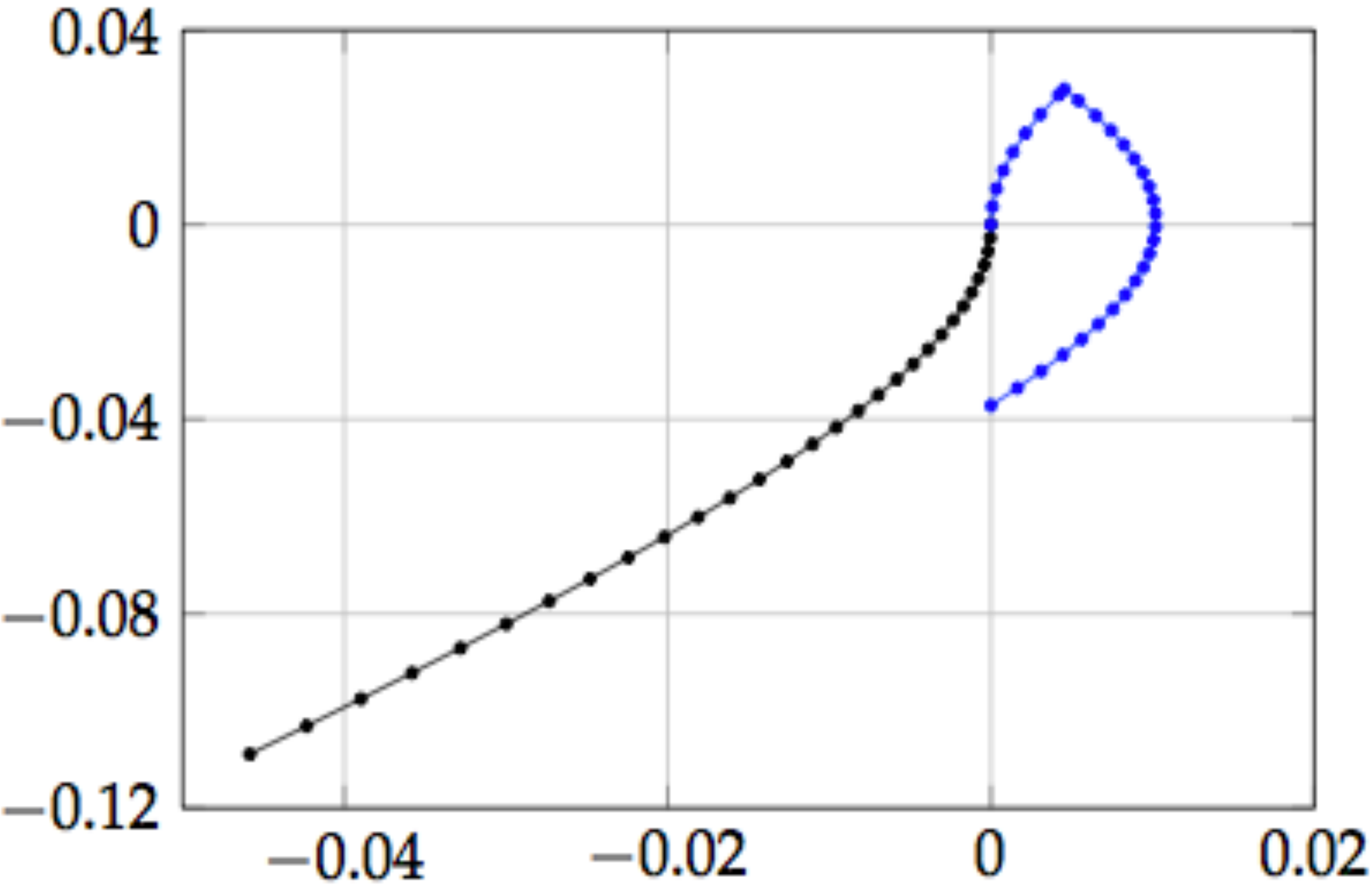


Optimize over initial $x(0)$



Commitment vs. discretion

Paths for $x(t), \pi(t)$. Commitment in blue, discretion in black



Commitment to inflation? Or boom?

- Krugman (1998) and older literature emphasizes importance of commitment to deliver *inflation*

- Werning (2012) argues that real goal is to deliver *boom* (though optimum generally features some positive inflation)

- Three devices to illustrate this point

  (i) completely rigid prices, $\kappa = 0$
  (ii) commitment to exit inflation $\pi(T)$ only
  (iii) exogenous constraint to avoid inflation

  In each case obtains result that commitment to $i(t) = 0$ after trap is over is motivated by desire to deliver a boom

Appendix: details for 'after the storm'

Phase III. After the storm

- Recall that at beginning of Phase III
  - initial $x(\hat{T}), \pi(\hat{T})$ are given (not free)
  - ZLB is not binding so $i(t) = I(\bar{r}, \pi(t))$
- Hence dynamic system given by

$$\dot{x}(t) = -\kappa\lambda\pi(t)$$

$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

- Guess $x(t) = \phi\pi(t)$ for some ϕ to be determined
- Hence $\dot{x}(t) = \phi\dot{\pi}(t)$ so

$$-\kappa\lambda\pi(t) = \phi\Big(\rho\pi(t) - \kappa\phi\pi(t)\Big)$$

Phase III. After the storm

- Since this must hold for all $\pi(t)$ we have the restriction

$$q(\phi) = \kappa\phi^2 - \rho\phi - \kappa\lambda = 0$$

- Solving for the roots of this quadratic

$$\phi_1, \phi_2 = \frac{\rho \pm \sqrt{\rho^2 + 4\kappa^2\lambda}}{2\kappa}$$

(one of which is positive, the other negative)

- We want these dynamics to take us *towards* $x(\infty) = \pi(\infty) = 0$, so we choose the positive solution

$$\phi = \frac{\rho + \sqrt{\rho^2 + 4\kappa^2\lambda}}{2\kappa} > \frac{\rho}{\kappa} > 0$$