Advanced Macroeconomics

Lecture 17: monetary economics, part five

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1st Semester 2019

This class

- New Keynesian model in continuous time
- The zero lower bound. Optimal monetary policy in a liquidity trap. Today discretionary policy, next class policy with commitment
- Further reading
  - ◇ Werning (2012): Managing a liquidity trap: Monetary and fiscal policy, MIT working paper, sections 1–3

Discrete time New Keynesian model

Three equations, without shocks

(1) Dynamic IS curve, in changes

$$\Delta\hat{x}_{t+1} = \frac{i_t - \hat{\pi}_t - r_t^n}{\sigma}$$

(2) New Keynesian Phillips curve, in changes

$$\Delta\hat{\pi}_{t+1} = (1-\beta)\hat{\pi}_{t+1} - \kappa\hat{x}_t$$

(3) Interest rate rule, say

$$i_t = r_t^n + \phi_\pi \hat{\pi}_t$$

Unique equilibrium if $\phi_\pi > 1$ but multiple equilibria if $\phi_\pi < 1$

Continuous time New Keynesian model

Three equations, without shocks

(1) Dynamic IS curve

$$\dot{x}(t) = \frac{i(t) - \pi(t) - r^n(t)}{\sigma}$$

(2) New Keynesian Phillips curve

$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

(3) Interest rate rule, say

$$i(t) = r^n(t) + \phi_\pi \pi(t),$$

Unique equilibrium if $\phi_\pi > 1$ but multiple equilibria if $\phi_\pi < 1$

Dynamics

- Qualitative dynamics, for phase diagram

$$\dot{x}(t) = \frac{1}{\sigma}(\phi_\pi - 1)\pi(t) > 0 \quad \Leftrightarrow \quad \begin{cases} \pi(t) > 0 & \text{if} \quad \phi_\pi > 1 \\ \pi(t) < 0 & \text{if} \quad \phi_\pi < 1 \end{cases}$$

$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t) > 0 \quad \Leftrightarrow \quad x(t) < (\rho/\kappa)\pi(t)$$

- Magnitude of ϕ_π determines *direction of $x(t)$ flow* and hence stability properties (eigenvalues) of system

- Initial values $x(0), \pi(0)$ both free — both output gap and inflation are jump variables

- Jump to $(x(0), \pi(0)) = (0, 0)$ if $\phi_\pi > 1$

Werning (2012)

- What is *optimal policy* if economy in liquidity trap?

- Welfare evaluated according to quadratic loss function

$$L = \frac{1}{2}\int_0^\infty e^{-\rho t}\left(x(t)^2 + \lambda\pi(t)^2\right)dt, \qquad \lambda \equiv \bar{\lambda}/\kappa \geq 0$$

  Weight on inflation $\lambda = \bar{\lambda}/\kappa \to 0$ as prices become fully flexible, $\kappa \to \infty$ (cf., 'microfoundations' of welfare weights in Lecture 15)

- Nominal interest rate $i(t)$ must satisfy the ZLB constraint

$$i(t) \geq 0$$

Liquidity trap of length T

- Suppose natural real rate takes form

$$r^n(t) = \begin{cases} \underline{r} & t \in [0, T) \\ \overline{r} & t \in [T, \infty) \end{cases} \qquad \text{with} \quad \underline{r} < 0 < \overline{r}$$

for some given horizon $T > 0$

- If $r^n(t) > 0$ for all t then can achieve $(x(t), \pi(t)) = (0, 0)$ with a sufficiently reactive interest rate rule

- But if $r^n(t) < 0$ for some t, economy stuck in a liquidity trap

Discretion

- Monetary authority cannot credibly commit to future actions
  - in other words, a *time-consistency* problem
- From $t \geq T$ monetary authority will try to do what is optimal from $t \geq T$ on irrespective of past announcements
- Hence from $t \geq T$ on, monetary authority will implement

$$(x(t), \pi(t)) = (0, 0) \qquad \text{for} \qquad t \geq T$$

(e.g., by sufficiently reactive interest rate rule)

- How does this affect dynamics *during* liquidity trap $t < T$?

Discretion

- System of differential equations with $i(t) = 0$ during trap $t < T$

$$\dot{x}(t) = -\sigma^{-1}(\pi(t) + \underline{r})$$
$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

with terminal condition

$$(x(T), \pi(T)) = (0, 0)$$

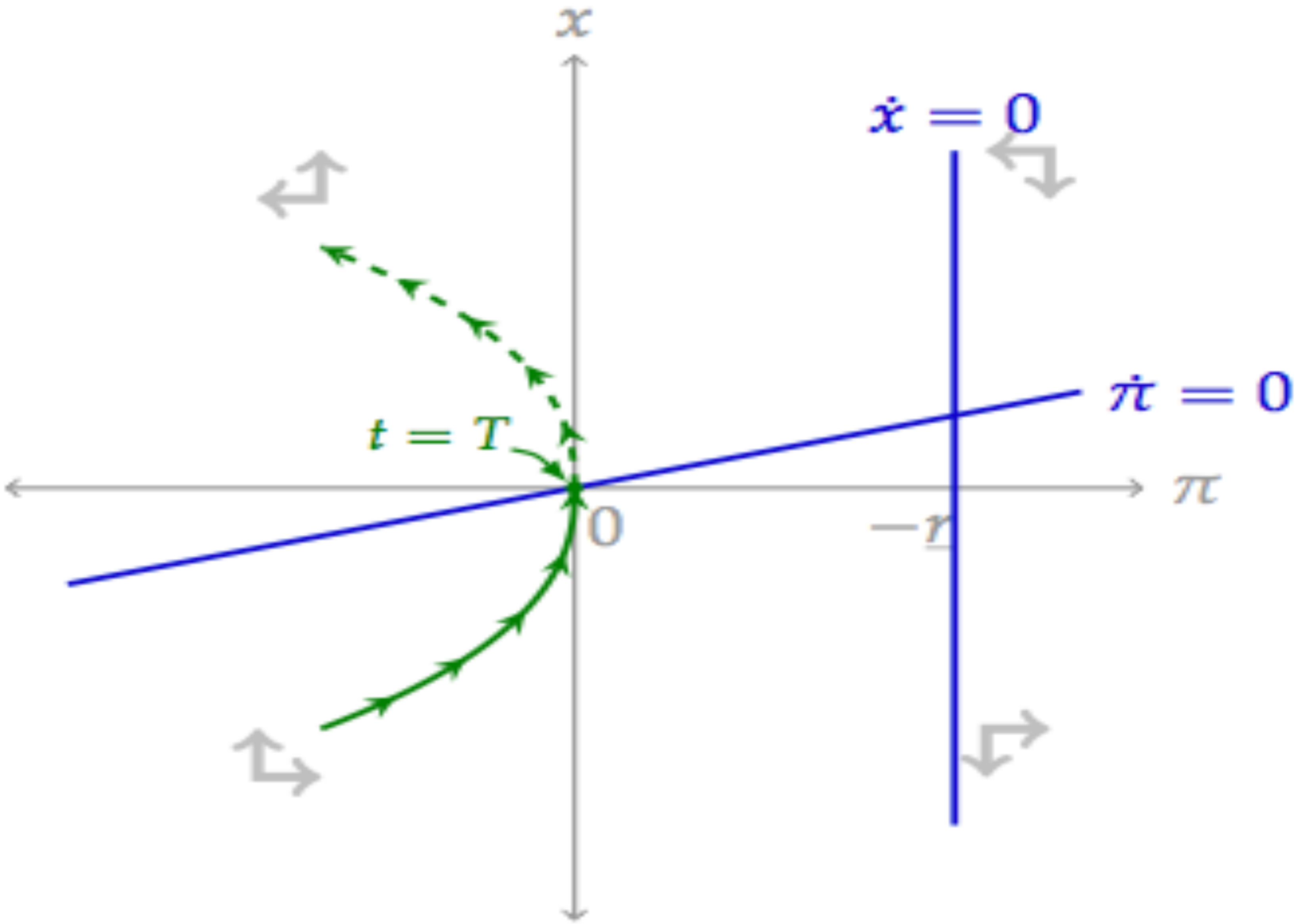
- Initial values $x(0), \pi(0)$ both free — both output gap and inflation are jump variables

- Qualitative dynamics, for phase diagram

$$\dot{x}(t) > 0 \quad \Leftrightarrow \quad \pi(t) < -\underline{r} > 0$$
$$\dot{\pi}(t) > 0 \quad \Leftrightarrow \quad x(t) < (\rho/\kappa)\pi(t)$$

Discretion

Before T, binding ZLB $i(t) = 0$. Obtains $x(T) = \pi(T) = 0$ at end of liquidity trap.



Deflation and recession

- Solution features *deflation*, $\pi(0) < 0$ and *recession*, $x(0) < 0$. Both gradually alleviated as $t \to T$

- Extent of initial recession is increasing in length of trap T

$$x(0), \pi(0) \to -\infty \qquad \text{as} \qquad T \to \infty$$

  (larger T means starting further from origin)

- Intuition: real interest rate $i(t) - \pi(t) = -\pi(t) > 0$ is *too high* during liquidity trap. Suppresses consumption and output, makes forward-looking inflation even lower, exacerbates problem

- Problem is inability to commit to actions *after* the liquidity trap, in particular inability to commit to other than $x(T) = \pi(T) = 0$

Harmful effects of price flexibility

- Surprisingly, outcomes worse if prices more flexible (high κ)

- High κ means a given $x(t) < 0$ creates more deflation $\pi(t) < 0$, making real rates even higher

- Euler equation then implies higher growth $\dot{x}(t)$ to reach $x(T) = 0$. But this means $x(0)$ must be even lower

- Perfectly rigid prices $\kappa = 0$ deliver a better outcome

- Benefits of price flexibility only obtained if monetary policy permits $\pi(t) > 0$ under some circumstances (which here it does not)

Higher inflation target?

- Consider a *suboptimal* policy that delivers steady state inflation

$$\pi(t) = \bar{\pi} \equiv -\underline{r} > 0, \quad \text{and} \quad x(t) = \bar{x} \equiv -\frac{\rho}{\kappa}\underline{r} > 0$$

(positive inflation, positive output gap for all $t \geq 0$)

- Commitment to higher inflation *after* the trap improves welfare

- Permanent sacrifice to solve a temporary problem. Large sacrifice if $\bar{\pi}$ high or T short etc, small sacrifice if prices flexible

- This is not optimal but approaches optimality as prices become fully flexible, $\kappa \to \infty$, since $\lambda = \bar{\lambda}/\kappa$ implies loss $L \to 0$

Appendix: deriving the continuous time limit

Deriving continuous time NK model

- Consumption Euler equation, without shocks

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} Q_t^{-1}$$

- Over a period of length $\Delta > 0$

$$(c_t \Delta)^{-\sigma} = \beta^{\Delta} (c_{t+\Delta} \Delta)^{-\sigma} \frac{P_t}{P_{t+\Delta}} Q_t^{-\Delta}$$

(all flows multiplied by period length)

- Taking logs, and rearranging

$$i_t \Delta = \rho \Delta + \sigma(\log c_{t+\Delta} - \log c_t) + (\log P_{t+\Delta} - \log P_t)$$

where $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$

Deriving continuous time NK model

- Hence

$$i(t) = \rho + \sigma \left(\frac{\log c(t+\Delta) - \log c(t)}{\Delta} \right) + \frac{\log P(t+\Delta) - \log P(t)}{\Delta}$$

- Take limit as $\Delta \to 0$ to get

$$\boxed{i(t) = \rho + \sigma \frac{\dot{c}(t)}{c(t)} + \pi(t)}$$

where

$$\frac{\dot{c}(t)}{c(t)} = \frac{d \log c(t)}{dt}$$

and

$$\pi(t) \equiv \frac{\dot{P}(t)}{P(t)} = \frac{d \log P(t)}{dt}$$

Dynamic IS curve

- Now let $y^n(t)$ denote natural output (flexible price output). Let $r^n(t)$ denote natural real rate, satisfies

$$r^n(t) = \rho + \sigma \frac{\dot{y}^n(t)}{y^n(t)}$$

- Let $x(t) \equiv \log y(t) - \log y^n(t)$ denote the log output gap. Since $c(t) = y(t)$ have $\dot{x}(t) = \dot{c}(t)/c(t) - \dot{y}^n(t)/y^n(t)$ and hence

$$\boxed{\dot{x}(t) = \frac{i(t) - \pi(t) - r^n(t)}{\sigma}}$$

This is the *dynamic IS curve* in continuous time

New Keynesian Phillips curve

- Similarly for new Keynesian Phillips curve, without shocks

$$\pi_t = \beta\pi_{t+1} + \kappa x_t$$

- Over a period of length $\Delta > 0$

$$\pi_t = \beta^{\Delta}\pi_{t+\Delta} + \kappa x_t \Delta$$

- Write in terms of change in inflation

$$\pi_{t+\Delta} - \pi_t = (1 - \beta^{\Delta})\pi_{t+\Delta} - \kappa x_t \Delta$$

New Keynesian Phillips curve

- Hence

$$\frac{\pi(t+\Delta)-\pi(t)}{\Delta} = \frac{1-\beta^{\Delta}}{\Delta}\pi(t+\Delta) - \kappa x(t)$$

- Take limit as $\Delta \to 0$ to get

$$\boxed{\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)}$$

This is the *new Keynesian Phillips curve* in continuous time

- Uses l'Hôpital's rule

$$\lim_{\Delta\to 0}\frac{1-\beta^{\Delta}}{\Delta} = \lim_{\Delta\to 0}\frac{-\beta^{\Delta}\log\beta}{1} = -\log\beta \equiv \rho$$

(or can use $\beta^{\Delta} = e^{-\rho\Delta} \approx 1-\rho\Delta$)

New Keynesian Phillips curve

- Integrating the new Keynesian Phillips curve forward

$$\pi(t) = \kappa \int_0^\infty e^{-\rho s} x(t+s)\, ds$$

(positive gaps increase inflation, negative gaps decrease it)