Advanced Macroeconomics

Lecture 16: monetary economics, part four

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This class

- Monetary policy tradeoffs in the new Keynesian model
- Optimal discretionary policy vs. optimal policy with commitment

Monetary policy tradeoffs

- In basic new Keynesian model, optimal policy seeks to replicate the flexible price equilibrium

- A version of *strict inflation targeting* applies, aggressive response to inflation stabilizes output gap as a byproduct

- This is because in basic new Keynesian model, the flexible price equilibrium is itself efficient

- In practice, monetary authorities face tradeoff between stabilizng inflation and stabilizing output gap (at least in short run)

- In the new Keynesian model, such tradeoffs arise when the flexible price equilibrium is *not* efficient (e.g., due to real rigidities)

Monetary policy tradeoffs

- Monetary authority seeks to minimize the loss function

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left(\hat{x}_t^2+\lambda\hat{\pi}_t^2\right)\right\},\qquad \lambda\equiv\frac{\omega_\pi}{\omega_x}>0$$

subject to modified new Keynesian Phillips curve

$$\hat{\pi}_t=\beta\mathbb{E}_t\left\{\hat{\pi}_{t+1}\right\}+\kappa\hat{x}_t+u_t$$

- '*Cost push shocks*' (more precisely, transitory deviations between flexible price equilibrium and efficient allocation)

$$u_{t+1}=\rho_u u_t+\varepsilon_{t+1},\qquad \varepsilon_{t+1}\sim \text{IID and } N(0,\sigma_u^2)$$

with persistence $0\leq\rho_u<1$

- Solution depends on ability to commit to future actions.
Here commitment power is exogenous, either have it or not

Discretion

- *Inability to commit to future actions*

- Because no endogenous state variable, problem essentially static

- Each period, monetary authority minimizes

$$\frac{1}{2}\left(\hat{x}_t^2 + \lambda \hat{\pi}_t^2\right)$$

subject to

$$\hat{\pi}_t = \kappa \hat{x}_t + \xi_t, \qquad \xi_t \equiv \beta \mathbb{E}_t \left\{\hat{\pi}_{t+1}\right\} + u_t$$

where ξ_t is taken as given

Discretion

- Optimal policy involves *leaning-against-the-wind*

$$\hat{x}_t = -\kappa\lambda\hat{\pi}_t$$

- A given change in $\hat{\pi}_t$ is associated with large change in $\hat{x}_t$ if
  - high λ, monetary authority concerned with inflation, or
  - high κ, prices are quite flexible

- Implies inflation satisfies the stochastic difference equation

$$\hat{\pi}_t = \frac{\beta}{1+\kappa^2\lambda}\mathbb{E}_t\left\{\hat{\pi}_{t+1}\right\} + \frac{1}{1+\kappa^2\lambda}u_t$$

- Solve for $\hat{\pi}_t$ in terms of u_t using method undetermined coeff.

Discretion

- Gives solution

$$\hat{\pi}_t = +\frac{1}{(1-\beta\rho_u)+\kappa^2\lambda}u_t, \qquad \hat{x}_t = -\frac{\kappa\lambda}{(1-\beta\rho_u)+\kappa^2\lambda}u_t$$

- Cost push increases $\hat{\pi}_t$, optimal policy stabilizes by reducing $\hat{x}_t$
  - if high weight on inflation, $\lambda \to \infty$, output does all the response

$$\hat{\pi}_t = 0, \qquad \hat{x}_t = -\frac{1}{\kappa}u_t$$

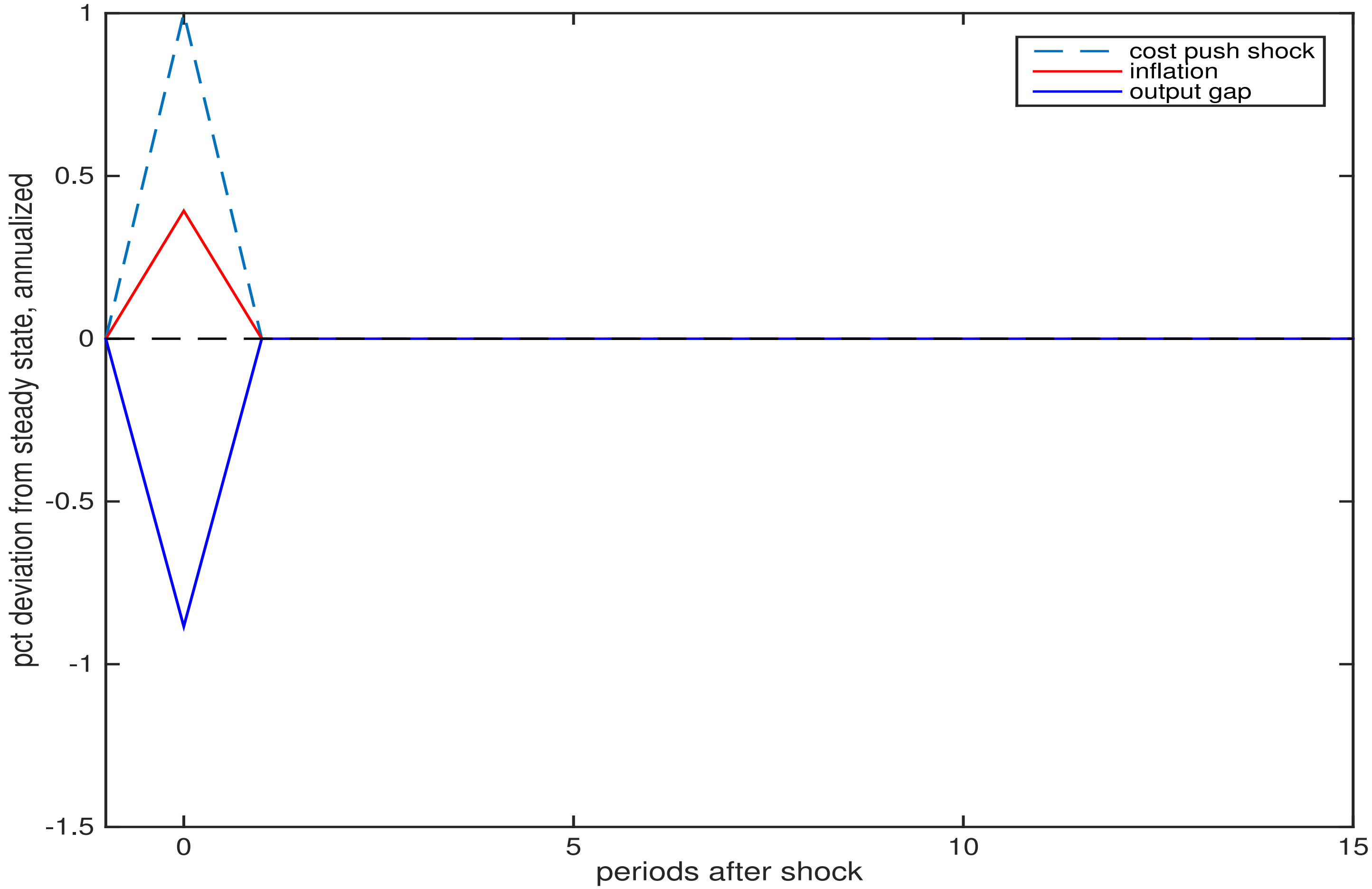
(then large $\hat{x}_t$ response if κ low — i.e., very sticky prices)

  - if high weight on output gap, $\lambda \to 0$, inflation does all the response

$$\hat{\pi}_t = +\frac{1}{1-\beta\rho_u}u_t, \qquad \hat{x}_t = 0$$

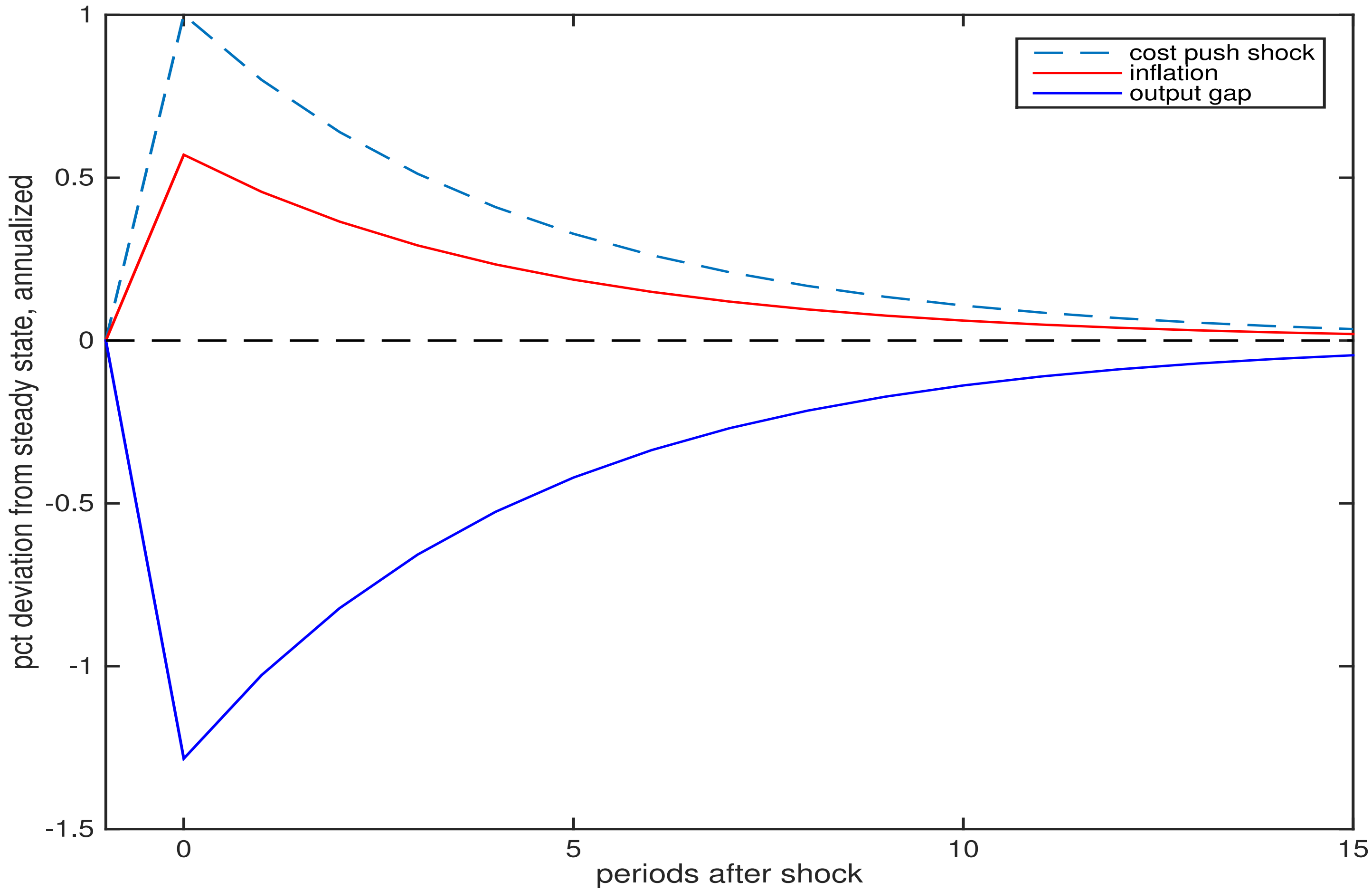
(then large $\hat{\pi}_t$ response if shock persistent; same if $\kappa = 0$)

Transitory cost push shock, $\rho_u = 0$



One-time cost push shock u_0 and implied paths of inflation $\hat{\pi}_t$ and output gap $\hat{x}_t$ under *discretion*. No endogenous persistence — inflation and output gap back at steady state as soon as shock is over.

Persistent cost push shock, $\rho_u \in (0, 1)$



Cost push shock $u_t = \rho_u^t u_0$ and implied paths of inflation $\hat{\pi}_t$ and output gap $\hat{x}_t$ under *discretion*. Inflation and output gap inherit persistence of the shock.

Interest rates

- From dynamic IS curve

$$r_t = r_t^n + \sigma \mathbb{E}_t \{\Delta \hat{x}_{t+1}\}$$

- Hence interest rates

$$i_t = r_t^n + \sigma \mathbb{E}_t \{\Delta \hat{x}_{t+1}\} + \mathbb{E}_t \{\hat{\pi}_{t+1}\}$$

- For this example, works out to

$$i_t = r_t^n + \left[\frac{(1-\rho_u)\sigma\kappa\lambda + \rho_u}{(1-\beta\rho_u) + \kappa^2\lambda} \right] u_t$$

so that cost push shocks increase the policy rate

Implementation

- Passive interest rate rule of the form

$$i_t = r_t^n + \left[\frac{(1-\rho_u)\sigma\kappa\lambda + \rho_u}{(1-\beta\rho_u) + \kappa^2\lambda}\right] u_t \equiv i_t^*$$

does not implement the optimal discretionary outcome
(since does not guarantee unique solution)

- Need active interest rate rule, such as

$$i_t = i_t^* + \phi_\pi(\hat{\pi}_t - \hat{\pi}_t^*), \qquad \phi_\pi > 1$$

where $\hat{\pi}_t^*$ is inflation rate that policy wants to implement

$$\hat{\pi}_t^* \equiv \frac{1}{(1-\beta\rho_u) + \kappa^2\lambda} u_t$$

(so rule specifies sufficiently reactive '*off-equilibrium threat*')

- Then indeed, in equilibrium, $\hat{\pi}_t = \hat{\pi}_t^*$ and $i_t = i_t^*$ etc

Instrument rules vs. targeting rules

- *Instrument rules* like like

  $$`\, i_t = i_t^* + \phi_\pi(\hat{\pi}_t - \hat{\pi}_t^*)\, '$$

  impractical, requires too much confidence in model

- Leads some practitioners to prefer *targeting rules* like

  $$`\, \hat{x}_t = -\kappa\lambda\hat{\pi}_t\, '$$

  which requires less confidence in model

- Even targeting rules like this need estimates of $\hat{x}_t$ in real time

Commitment

- Now monetary authority solves genuinely dynamic problem

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\hat{x}_t^2 + \lambda \hat{\pi}_t^2 \right) \right\}, \qquad \lambda > 0$$

  subject to the forward-looking constraint

$$\hat{\pi}_t = \beta \mathbb{E}_t \left\{ \hat{\pi}_{t+1} \right\} + \kappa \hat{x}_t + u_t$$

- Monetary authority can now make intertemporal tradeoffs

Monetary authority's problem

- Lagrangian with multiplier μ_t for each constraint

$$\mathcal{L} = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\hat{x}_t^2 + \lambda \hat{\pi}_t^2 \right) + \mu_t \left(\hat{\pi}_t - \beta \hat{\pi}_{t+1} - \kappa \hat{x}_t - u_t \right) \right] \right\}$$

- Some key first order conditions

$$\hat{x}_t : \qquad \hat{x}_t - \mu_t \kappa = 0$$

$$\hat{\pi}_t : \qquad \lambda \hat{\pi}_t + \mu_t - \mu_{t-1} = 0$$

$$\mu_t : \qquad \hat{\pi}_t - \beta \mathbb{E}_t \left\{ \hat{\pi}_{t+1} \right\} - \kappa \hat{x}_t - u_t = 0$$

These hold at every date and state

Monetary authority's problem

- On impact

$$\hat{x}_0 = +\kappa\mu_0 = -\kappa\lambda\hat{\pi}_0$$

(since $\mu_{-1} = 0$, only constrained for $t \geq 0$)

- Thereafter

$$\Delta\hat{x}_t = +\kappa\Delta\mu_t = -\kappa\lambda\hat{\pi}_t$$

(multipliers follow random walk with increment $\Delta\mu_t = -\lambda\hat{\pi}_t$)

- Since price level is cumulative inflation, we have

$$\hat{x}_t = -\kappa\lambda\hat{p}_t, \qquad \hat{p}_t = \sum_{k=0}^{t} \hat{\pi}_k$$

Solving the model with commitment

- Write new Keynesian Phillips curve in terms of price level

$$\hat{p}_t - \hat{p}_{t-1} = \beta \mathbb{E}_t \left\{ \hat{p}_{t+1} - \hat{p}_t \right\} + \kappa \hat{x}_t + u_t$$

- Substitute in output gap in terms of price level, $\hat{x}_t = -\kappa\lambda\hat{p}_t$, and rearrange to get stochastic difference equation

$$\hat{p}_t = \gamma \hat{p}_{t-1} + \gamma \beta \mathbb{E}_t \left\{ \hat{p}_{t+1} \right\} + \gamma u_t$$

where, for short,

$$\gamma \equiv \frac{1}{1 + \beta + \kappa^2 \lambda} \in (0, 1)$$

- Now guess

$$\hat{p}_t = \psi_{pp} \hat{p}_{t-1} + \psi_{pu} u_t$$

and solve for ψ_{pp} and ψ_{pu} using method undetermined coeff.

Solving the model with commitment

- Stable root from quadratic in ψ_{pp}, governs endogenous dynamics

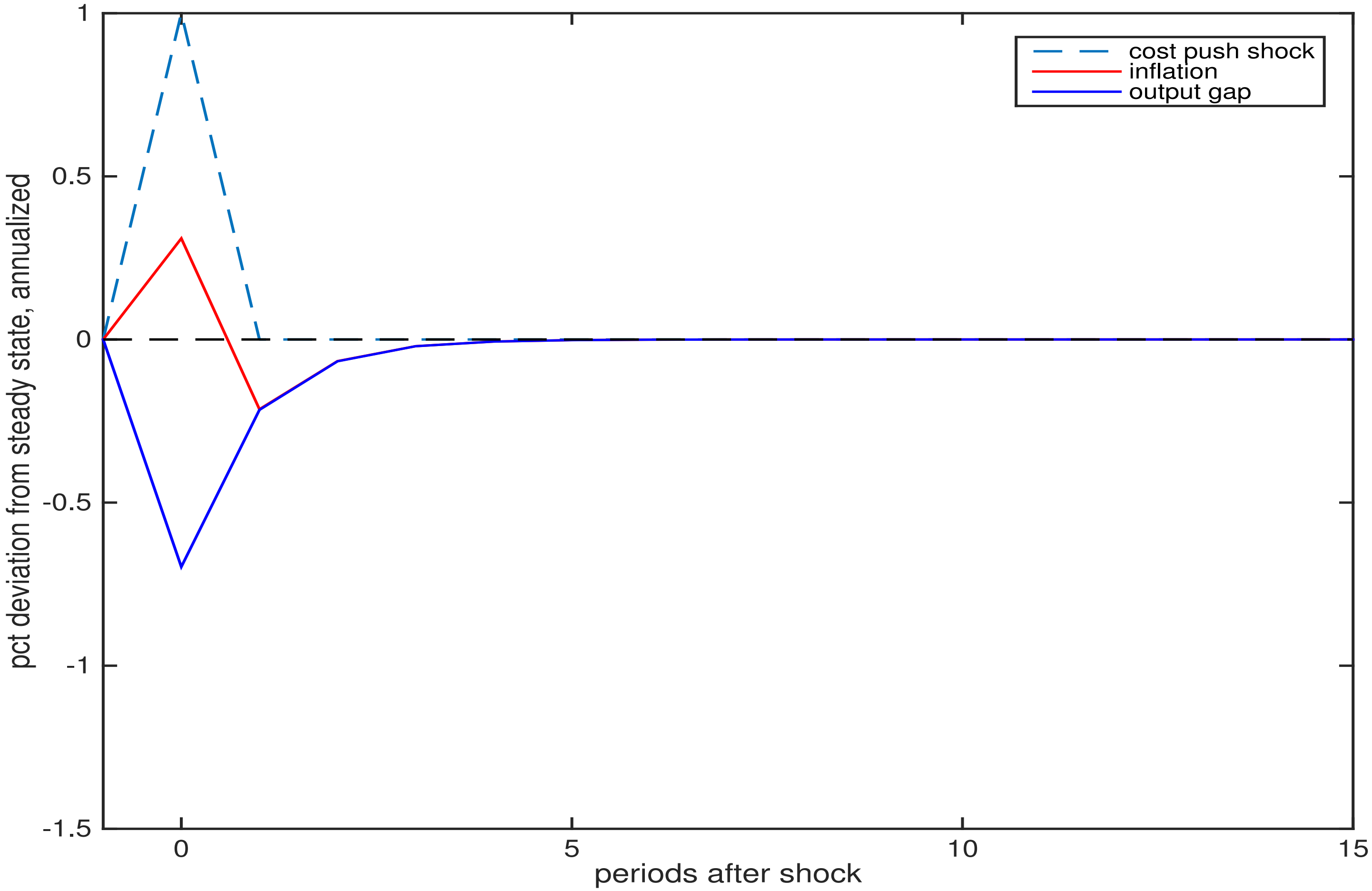
$$\psi_{pp} = \frac{1 - \sqrt{1 - 4\gamma^2\beta}}{2\gamma\beta} \in (0, 1)$$

- Response to cost push shock

$$\psi_{pu} = \frac{\psi_{pp}}{1 - \psi_{pp}\beta\rho_u} > 0$$

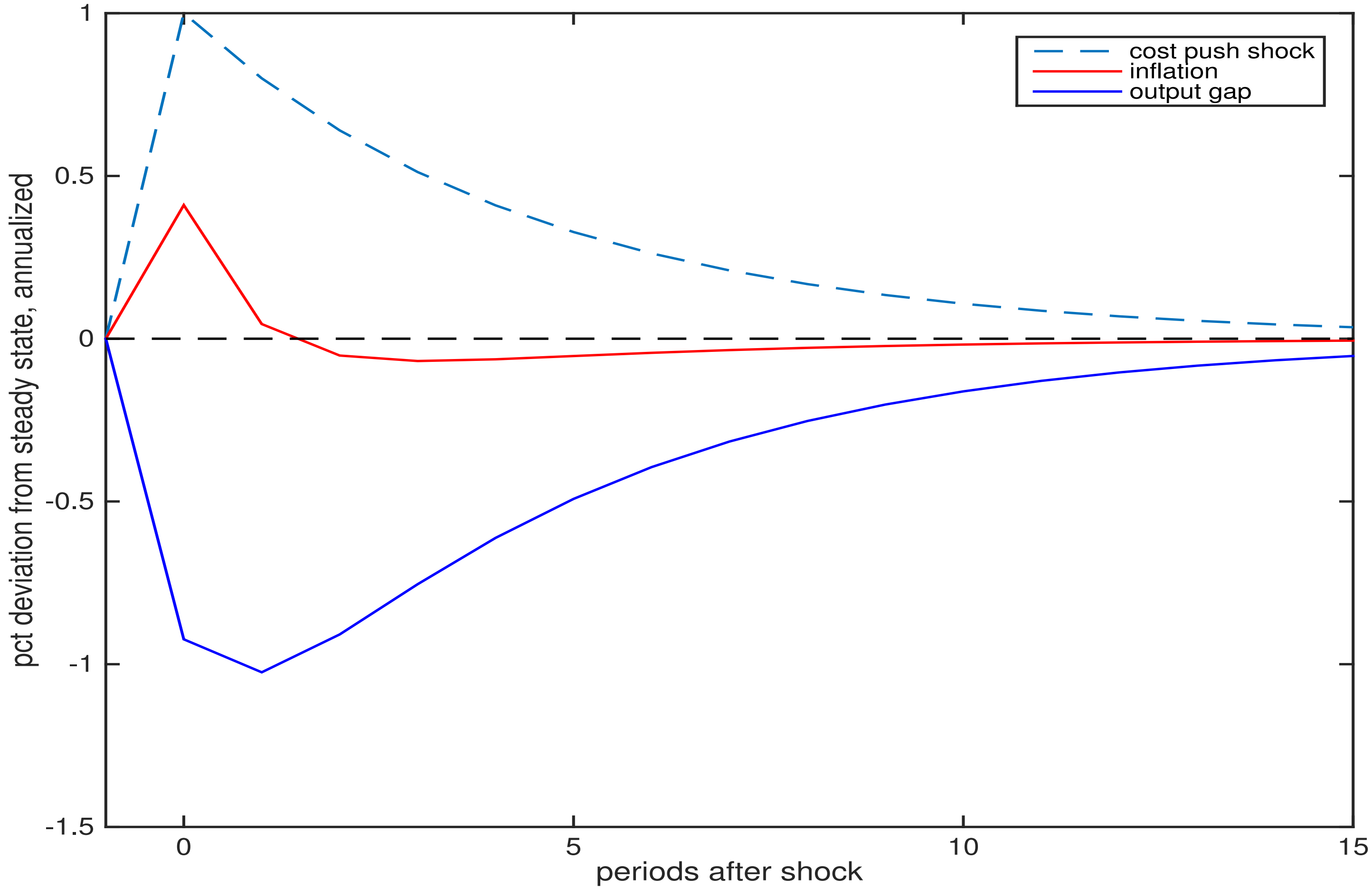
- Then recover output gap $\hat{x}_t = -\kappa\lambda\hat{p}_t$, inflation $\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$ etc

Transitory cost push shock, $\rho_u = 0$



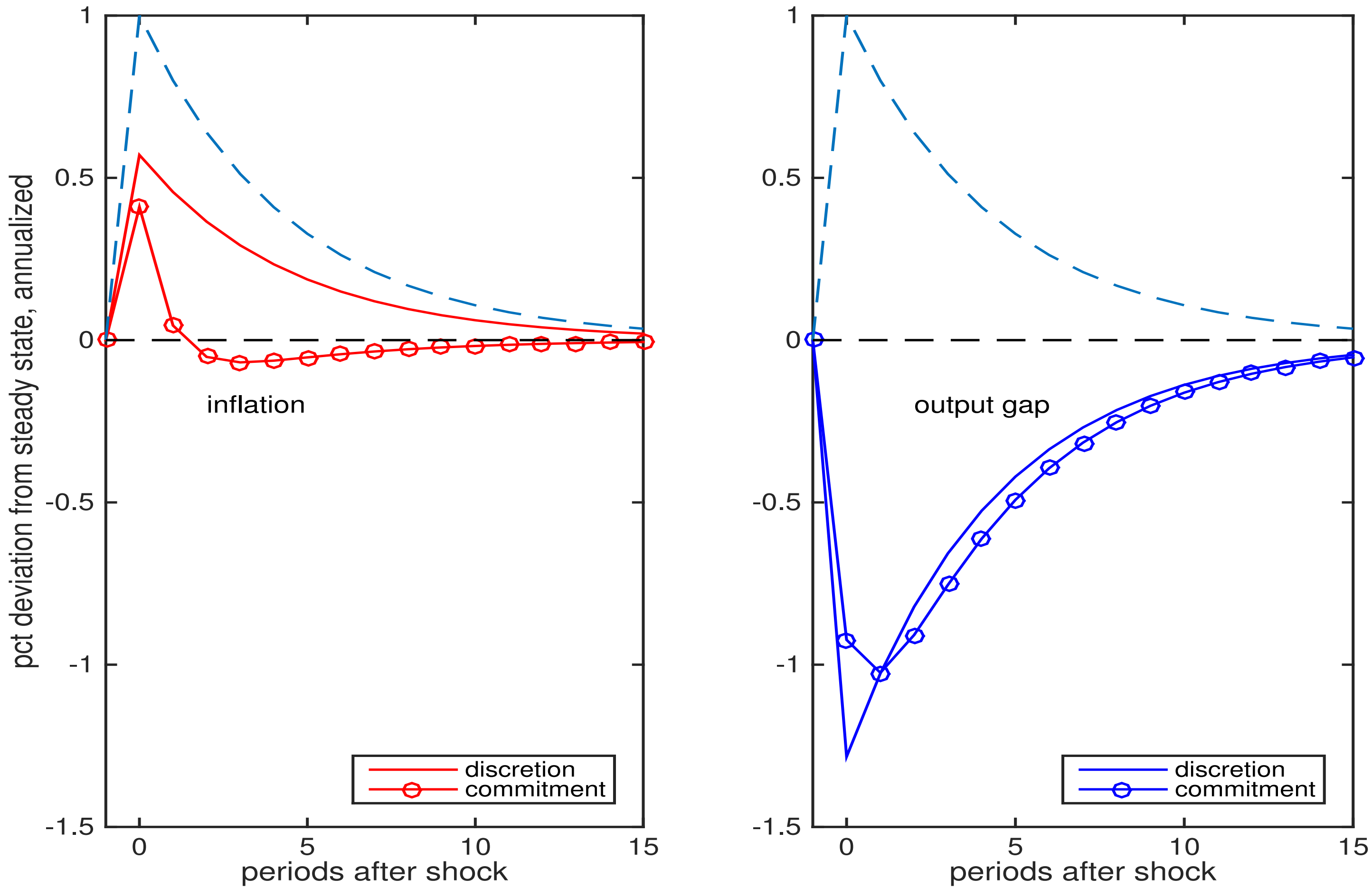
One-time cost push shock u_0 and implied paths of inflation $\hat{\pi}_t$ and output gap $\hat{x}_t$ under *commitment*. Endogenous persistence — inflation and output gap continue to depart from steady state even when shock is over. Inflation overshoots steady state.

Persistent cost push shock, $\rho_u \in (0,1)$



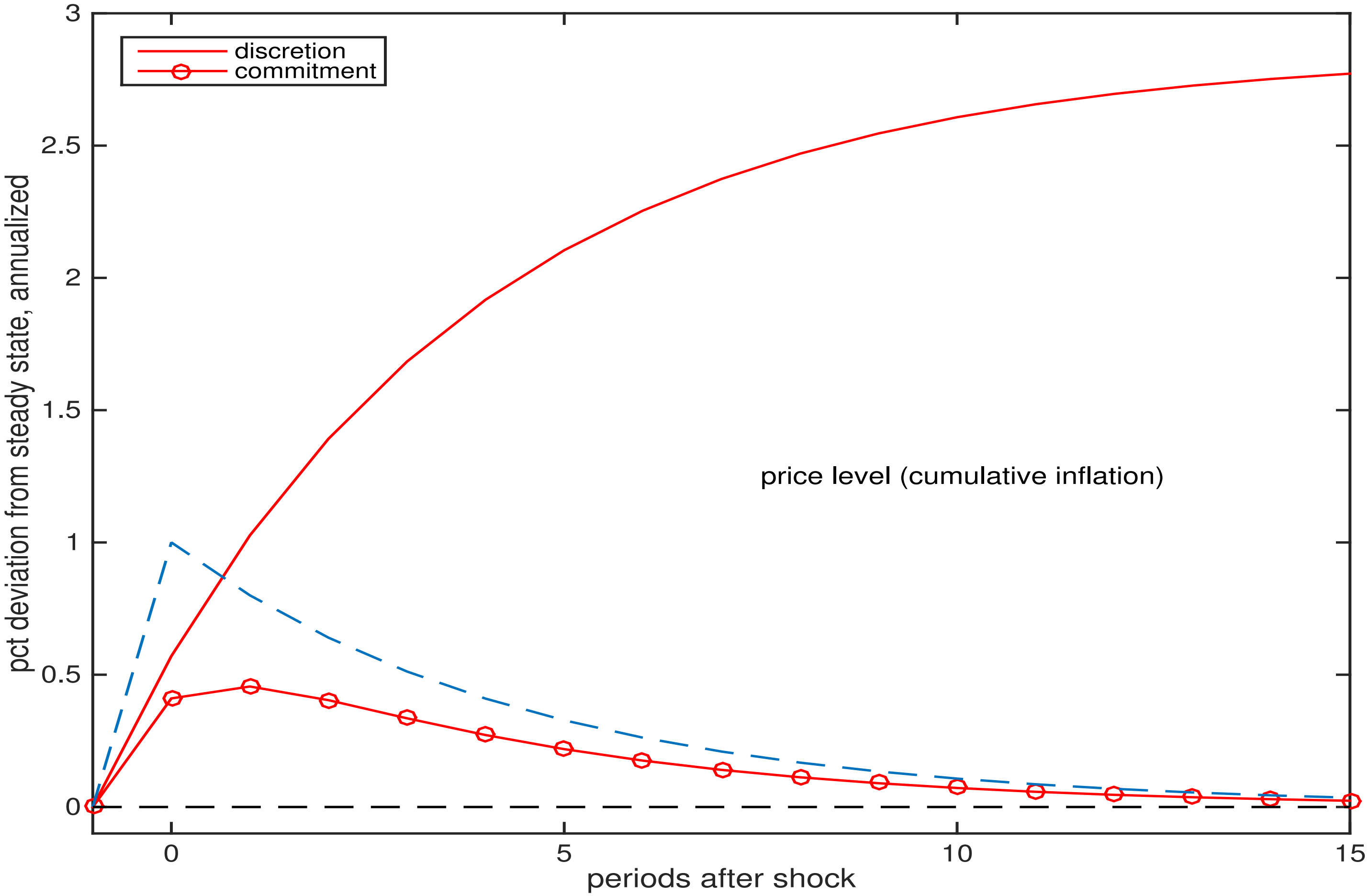
Cost push shock $u_t = \rho_u^t u_0$ and implied paths of inflation $\hat{\pi}_t$ and output gap $\hat{x}_t$ under *commitment*. Output gap continues to fall, $\Delta\hat{x}_t < 0$ while $\hat{\pi}_t > 0$. Then recovers back to steady state $\Delta\hat{x}_t > 0$ with $\hat{\pi}_t < 0$.

Discretion vs. commitment



Under commitment, smaller impact effects on inflation and output gap, inflation overshoots steady state. Under commitment, policy trades off larger output gaps in future in return for smaller output gap on impact, lower overall volatility.

Discretion vs. commitment



Under discretion, inflation always positive so permanent increase in price level $\hat{p}_t = \sum_{k=0}^{t} \hat{\pi}_k$. Under commitment, period of negative inflation offsets initial period of positive inflation so that $\hat{p}_t$ returns to initial level.

Implementation

- Can implement optimal policy under commitment with active rule

$$i_t = i_t^* + \phi_p(\hat{p}_t - \hat{p}_t^*), \qquad \phi_p > 0$$

where i_t^* denotes equilibrium interest rate (from dynamic IS curve) and $\hat{p}_t^*$ denotes price level that policy wants to implement

- Any $\phi_p > 0$ suffices to ensure unique solution

- Then, in equilibrium, $i_t = i_t^*$ and $\hat{p}_t = \hat{p}_t^*$ etc

Stabilization bias

- Under discretion, policy stabilizes output gap myopically
- Does not make intertemporal tradeoffs in output gap stabilization, does not trade larger future output gaps for smaller gap on impact
- This is known as *stabilization bias* under discretion