Advanced Macroeconomics

Lecture 15: monetary economics, part three

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This class

- Optimal policy in the basic new Keynesian model
- Implementing optimal policy, equilibrium stability and uniqueness
- Evaluating simple policy rules

Suboptimality in the new Keynesian model

Two sources of suboptimality

(i) *monopolistic competition* (unrelated to sticky prices)

market power, firms set markup over marginal cost

(ii) *sticky prices*

inefficient cross-sectional dispersion in relative prices

fluctuations in average markup

Optimal policy in the new Keynesian model

- Suppose underlying flexible price equilibrium is efficient (e.g., employment subsidy to correct market power distortion)

- Suppose initial condition $P_{-1}(j) = P_{-1}$ for all firms j (i.e., no inherited inefficient relative price dispersion)

- If for $t = 0, 1, \ldots$ policy is somehow such that $P_t^* = P_{t-1}$ for all producers that get the opportunity, then

  - price level stabilized, $P_t = P_{t-1}$

  - average markup stabilized

  - no relative price dispersion going forward

Optimal policy in the new Keynesian model

- In short, such a policy achieves

$$\hat{y}_t = \hat{y}_t^n \qquad \Leftrightarrow \qquad \hat{x}_t = 0$$

$$\hat{\pi}_t = 0$$

$$i_t = r_t^n$$

- Output itself is not stabilized, $\hat{y}_t$ fluctuates 1-for-1 with $\hat{y}_t^n$ so that there are no fluctuations in the output gap $\hat{x}_t$

- Price stability, but not because valued for own sake, but rather to eliminate distortions caused by sticky prices

Intuition

- Such a policy implies that producers do not adjust prices even when given opportunity

- That is, Calvo constraint on price-setting is *not binding*

- Hence allocation coincides with flexible price allocation, which, given employment subsidy, is efficient

‘Divine coincidence’

- Does not require monetary authority to know or care about $\hat{y}_t^n$
- Policy that achieves $\hat{\pi}_t = 0$ delivers $\hat{y}_t = \hat{y}_t^n$ as byproduct
- Suggests monetary authority should focus on price stability?
  No trade-off between output gap and inflation stabilization?

Implementing optimal policy

- So how is such an outcome *implemented*?
  - importance of the 'Taylor principle'
  - eigenvalues revisited

Eigenvalues revisited

- Consider system of linear difference equations

$$\boldsymbol{x}_{t+1} = \boldsymbol{A}\boldsymbol{x}_t$$

- For unique solution, need

$$\begin{aligned} & \text{same number of stable roots as given initial conditions} \\ \Leftrightarrow\ & \text{same number of unstable roots as } \textit{missing} \text{ initial conditions} \end{aligned}$$

- Example: optimal growth model
  - two roots, one stable and one unstable
  - one given initial condition (capital stock)
  - hence unique solution

New Keynesian model

- *Deterministic dynamics* (shutting down shocks)

$$\begin{pmatrix} \hat{x}_{t+1} \\ \hat{\pi}_{t+1} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \hat{x}_t \\ \hat{\pi}_t \end{pmatrix}$$

  for some coefficient matrix $\boldsymbol{A}$

- Both $\hat{x}_t$ and $\hat{\pi}_t$ are 'jump' variables, no given initial condition

- Since two variables and two missing initial conditions, need *both* roots of $\boldsymbol{A}$ to be *unstable* (magnitude > 1)

New Keynesian model

- Non-policy block of the model

$$\hat{x}_t = -\frac{1}{\sigma}\left(i_t - \hat{\pi}_{t+1} - r_t^n\right) + \hat{x}_{t+1}$$

and

$$\hat{\pi}_t = \beta \hat{\pi}_{t+1} + \kappa \hat{x}_t$$

- Optimal policy requires

$$\begin{aligned} \hat{\pi}_t &= 0 \\ \hat{x}_t &= 0 \\ i_t &= r_t^n \end{aligned}$$

- Let's try and implement this with the rule $i_t = r_t^n$

Passive rule $i_t = r_t^n$

- With $i_t = r_t^n$, deterministic dynamics are given by

$$\begin{pmatrix} \hat{x}_{t+1} \\ \hat{\pi}_{t+1} \end{pmatrix} = \frac{1}{\beta} \begin{pmatrix} \beta + \frac{\kappa}{\sigma} & -\frac{1}{\sigma} \\ -\kappa & 1 \end{pmatrix} \begin{pmatrix} \hat{x}_t \\ \hat{\pi}_t \end{pmatrix}$$

- What are the eigenvalues of the coefficient matrix?

$$\boldsymbol{A} = \frac{1}{\beta} \begin{pmatrix} \beta + \frac{\kappa}{\sigma} & -\frac{1}{\sigma} \\ -\kappa & 1 \end{pmatrix}$$

Passive rule $i_t = r_t^n$

- Determinant

$$\det(\boldsymbol{A}) = \left(\frac{1}{\beta}\right)^2 \det\begin{pmatrix} \beta + \frac{\kappa}{\sigma} & -\frac{1}{\sigma} \\ -\kappa & 1 \end{pmatrix} = \frac{1}{\beta} > 1$$

- Trace

$$\operatorname{tr}(\boldsymbol{A}) = \left(\frac{1}{\beta}\right) \operatorname{tr}\begin{pmatrix} \beta + \frac{\kappa}{\sigma} & -\frac{1}{\sigma} \\ -\kappa & 1 \end{pmatrix} = 1 + \frac{\kappa}{\beta\sigma} + \frac{1}{\beta} > 2$$

- Polynomial at unity

$$p(1) = 1 - \operatorname{tr}(\boldsymbol{A}) + \det(\boldsymbol{A}) = -\frac{\kappa}{\beta\sigma} < 0$$

- Implications

(i) product positive, so both eigenvalues have same sign
(ii) sum is positive, therefore from (i) both positive
(iii) polynomial $p(1) < 0$, eigenvalues not on same side of $+1$

$$0 < \lambda_1 < 1 < \lambda_2$$

Passive rule $i_t = r_t^n$

- Coefficient matrix $\boldsymbol{A}$ has $\lambda_1 < 1$ and $\lambda_2 > 1$
- But for unique solution, needed both roots to be unstable
- There are multiple solutions, i.e., *multiple equilibria*
  - one dimensional degree of indeterminacy
  - no reason to believe optimal outcome will emerge
- What about other rules? Can they 'reliably' implement optimum?

Active rule

- Now consider *active* interest rate rule

$$i_t = r_t^n + \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t$$

- Implies deterministic dynamics

$$\begin{pmatrix} \hat{x}_{t+1} \\ \hat{\pi}_{t+1} \end{pmatrix} = \frac{1}{\sigma\beta} \begin{pmatrix} \kappa + \beta(\sigma + \phi_x) & \beta\phi_\pi - 1 \\ -\sigma\kappa & \sigma \end{pmatrix} \begin{pmatrix} \hat{x}_t \\ \hat{\pi}_t \end{pmatrix}$$

- Now what are the eigenvalues? How do they depend on the policy coefficients ϕ_π, ϕ_x ?

Active rule

- Determinant

$$\det(\boldsymbol{A}) = \left(\frac{1}{\sigma\beta}\right)^2 \det\begin{pmatrix} \kappa + \beta(\sigma + \phi_x) & \beta\phi_\pi - 1 \\ -\sigma\kappa & \sigma \end{pmatrix}$$

$$= \frac{1}{\sigma\beta}\left(\sigma + \phi_x + \kappa\phi_\pi\right) > \frac{1}{\beta} > 1$$

- Trace

$$\mathrm{tr}(\boldsymbol{A}) = \left(\frac{1}{\sigma\beta}\right) \mathrm{tr}\begin{pmatrix} \kappa + \beta(\sigma + \phi_x) & \beta\phi_\pi - 1 \\ -\sigma\kappa & \sigma \end{pmatrix}$$

$$= \frac{1}{\sigma\beta}\left(\sigma + \kappa + \beta(\sigma + \phi_x)\right) > \frac{1}{\beta} + 1 > 2$$

Active rule

- Polynomial at unity

$$p(1) = 1 - \mathrm{tr}(\boldsymbol{A}) + \det(\boldsymbol{A}) = \frac{\phi_x(1-\beta) + \kappa(\phi_\pi - 1)}{\sigma\beta}$$

- Summary

  (i) product positive, so both eigenvalues have same sign
  (ii) sum is positive, therefore from (i) both positive
  (iii) therefore both eigenvalues > 1 if and only if $p(1) > 0$

- Necessary and sufficient condition for unique solution is therefore

$$\boxed{\phi_x(1-\beta) + \kappa(\phi_\pi - 1) > 0}$$

- In short, interest rate response needs to be '*sufficiently reactive*'

Taylor principle

- Intuition. In steady state

$$\frac{\partial i}{\partial \hat{\pi}} = \phi_\pi + \phi_x \frac{\partial \hat{x}}{\partial \hat{\pi}}, \qquad \frac{\partial \hat{x}}{\partial \hat{\pi}} = \frac{1-\beta}{\kappa}$$

- Therefore

$$\frac{\partial i}{\partial \hat{\pi}} > 1 \qquad \Leftrightarrow \qquad \phi_\pi + \phi_x \frac{1-\beta}{\kappa} > 1$$

- This is the same as our condition from the polynomial at unity

- Real rate rises in response to permanent increase in inflation

Equilibrium outcome

- Suppose $\phi_\pi > 1$ so that condition is satisfied
- In equilibrium, both endogenous variables jump to stable solution

$$\hat{x}_t = 0 \qquad \text{and} \qquad \hat{\pi}_t = 0$$

- Therefore

$$i_t = r_t^n$$

- But an *equilibrium outcome*, not a description of the *policy rule*
- '*Off-equilibrium threat*' of sufficient reaction
- Under these conditions, feedback rule implements optimal outcome. Other rules can also implement optimal outcome.

$$i_t = r_t^n + \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t$$

- Easier said than done. Practical shortcomings
  - requires knowledge of 'true structure' of economy (to compute natural output y_t^n,r_t^n etc), for which we need
  - all functional forms, all parameter values, shock realizations, etc
- Alternative is to look at *simple rules* that make policy instrument a function of observable variables only
- Goal then is to find rules that are 'robust' across many models, since no model is true (though some may be useful)
- Evaluate proposed rules according to a *welfare function*

Welfare function

- Representative household has utility

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right\}$$

- Taking 2nd order approximation, can be written in the form

$$-\frac{1}{2}\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\omega_x \, \hat{x}_t^2 + \omega_\pi \, \hat{\pi}_t^2 \right] \right\}$$

with weights ω_x, ω_π that depend on structural parameters

Welfare function

- Average *loss per period* is

$$\frac{1}{2}\left\{ \omega_x \operatorname{Var}\left[\, \hat{x}_t \,\right] + \omega_\pi \operatorname{Var}\left[\, \hat{\pi}_t \,\right] \right\}$$

- For our basic new Keynesian model, the weights work out to be

$$\omega_x = \sigma + \varphi$$

(from curvature parameters in utility function)

$$\omega_\pi = \frac{\varepsilon\theta}{(1-\theta)(1-\theta\beta)}$$

(increasing in price stickiness θ and in ε)

Interest rate rule

- *Simple rule*, not conditional on output gap or natural rate

$$\begin{aligned} i_t &= \rho + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \\ &= \rho + \phi_\pi \hat{\pi}_t + \phi_y \hat{x}_t + v_t \end{aligned}$$

  Implies 'shock' is $v_t \equiv \phi_y \hat{y}_t^n$, not a monetary policy shock

- Compute equilibrium outcomes, evaluate according to loss function for various settings of ϕ_π, ϕ_y

- Report losses as *percentages of steady state consumption*

Interest rate rule

- *Conditional on productivity shocks* (standard deviation = 1%)

ϕ_π	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1
Std($\hat{y}_t$)	1.85	2.07	2.25	1.06
Std($\hat{x}_t$)	0.44	0.21	0.03	1.23
Std($\hat{\pi}_t$)	0.69	0.34	0.05	1.94
Loss, %	1.02	0.25	0.006	7.98

- Large ϕ_π reduces both $\hat{x}_t$ and $\hat{\pi}_t$ volatility
  Large ϕ_y reduces $\hat{y}_t$ volatility but increases $\hat{x}_t, \hat{\pi}_t$ volatility

Interest rate rule

- *Conditional on demand shocks* (standard deviation = 1%)

ϕ_π	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1
Std($\hat{y}_t$)	0.59	0.68	0.28	0.31
Std($\hat{x}_t$)	0.59	0.68	0.28	0.31
Std($\hat{\pi}_t$)	0.20	0.23	0.09	0.10
Loss, %	0.10	0.13	0.02	0.02

- Large ϕ_π reduces both $\hat{y}_t, \hat{x}_t$ and $\hat{\pi}_t$ volatility
  Large ϕ_y reduces $\hat{y}_t$ and $\hat{x}_t$ volatility