Advanced Macroeconomics

Lecture 14: monetary economics, part two

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This class

- The basic new Keynesian model
- Sticky prices, output gaps, and the new Keynesian Phillips curve
- Equilibrium dynamics and response to shocks

Sticky prices

- Discrete time version of Calvo (1983)

- Firms have IID random opportunities to change prices

  - with probability θ, firm keeps current price
  - with probability $1 - \theta$, firm gets to re-optimize price

- Implies average duration of price is $1/(1 - \theta)$ periods

Law of motion for price level

- Recall ideal price index

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}$$

- Fraction θ of firms stuck with their old prices, fraction $1 - \theta$ re-optimize and choose P_t^* (by symmetry). Implies

$$P_t = \left(\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*\,1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

- Log-linearization around zero-inflation steady state

$$\hat{p}_t = \theta \hat{p}_{t-1} + (1-\theta) \hat{p}_t^*$$

or equivalently, in terms of inflation

$$\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1} = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1}) \tag{1}$$

Expected discounted profits

- Now need to determine reset price P_t^*

- Let $V_{t+k}(P_t^*)$ be nominal profit in period $t+k$ given P_t^* set at t

- Expected discounted nominal profits

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \mathcal{Q}_{t,t+k} V_{t+k}(P_t^*) \right\}$$

where $\mathcal{Q}_{t,t+k}$ is the nominal '*stochastic discount factor*'

$$\mathcal{Q}_{t,t+k} = \beta^k \frac{u_{c,t+k}}{u_{c,t}} \frac{P_t}{P_{t+k}}$$

Expected discounted profits

- Nominal profit in period $t+k$ given price P_t^* set at t

$$V_{t+k}(P_t^*) = \left(P_t^* - \frac{W_{t+k}}{z_{t+k}}\right)\left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} y_{t+k}$$

- Choose single number P_t^* to maximize

$$\mathbb{E}_t\left\{\sum_{k=0}^{\infty}\theta^k \mathcal{Q}_{t,t+k} V_{t+k}(P_t^*)\right\}$$

- First order condition

$$\mathbb{E}_t\left\{\sum_{k=0}^{\infty}\theta^k \mathcal{Q}_{t,t+k}\, V'_{t+k}(P_t^*)\right\} = 0$$

Optimal reset price

- Manipulating this first order condition gives

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \, u_{c,t+k} \, P_{t+k}^{\varepsilon-1} \, y_{t+k} \, \frac{W_{t+k}}{z_{t+k}} \right\}}{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \, u_{c,t+k} \, P_{t+k}^{\varepsilon-1} \, y_{t+k} \right\}}$$

- Reduces to usual markup formula $P_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{W_t}{z_t}$ if prices fully flexible (that is, if $\theta = 0$)

Optimal reset price

- Log-linear approximation (around zero-inflation steady state)

$$\hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k[\hat{w}_{t+k} - \hat{z}_{t+k} + \hat{p}_{t+k}]\right\}$$

where $\hat{w}_t - \hat{z}_t$ is the log dev. of *real* marginal cost w/z so that $\hat{w}_t - \hat{z}_t + \hat{p}_t$ is the log dev. of *nominal* marginal cost $W/z = wP/z$

- In recursive form, $\hat{p}_t^*$ solves stochastic difference equation

$$\hat{p}_t^* = (1-\theta\beta)[\hat{w}_t - \hat{z}_t + \hat{p}_t] + \theta\beta\mathbb{E}_t\left\{\hat{p}_{t+1}^*\right\} \qquad (2)$$

Inflation

- Combining (1) and (2), inflation solves

$$\hat{\pi}_t = \lambda[\hat{w}_t - \hat{z}_t] + \beta \mathbb{E}_t \left\{ \hat{\pi}_{t+1} \right\}, \qquad \lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$$

  Current inflation depends on current real marginal costs and expected future inflation

- Iterating forward

$$\hat{\pi}_t = \lambda \, \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \beta^k [\hat{w}_{t+k} - \hat{z}_{t+k}] \right\}$$

  Current inflation discounted sum of expected real marginal costs

- Common to rewrite real marginal cost in terms of an *output gap*

Output and real marginal cost

- Use household labor supply condition to write real marginal cost

$$\frac{w}{z} = \frac{l^{\varphi} c^{\sigma}}{z}$$

and use production function and goods market clearing to write

$$\frac{w}{z} = y^{\sigma+\varphi} \, z^{-(1+\varphi)}$$

- Hence in log deviations real marginal cost is

$$\hat{w}_t - \hat{z}_t = (\sigma + \varphi)\hat{y}_t - (1 + \varphi)\hat{z}_t$$

Natural output and output gap

- But recall from last lecture that if prices are fully flexible

$$\hat{y}_t^n = \psi_{yz}\hat{z}_t = \frac{1+\varphi}{\sigma+\varphi}\,\hat{z}_t$$

where $\hat{y}_t^n$ denotes the '*natural level of output*'

- So we can write

$$\hat{w}_t - \hat{z}_t = (\sigma+\varphi)(\hat{y}_t - \hat{y}_t^n)$$

In short, *real marginal cost is proportional to the output gap*

$$\hat{w}_t - \hat{z}_t = (\sigma+\varphi)\hat{x}_t, \qquad \hat{x}_t \equiv \hat{y}_t - \hat{y}_t^n$$

New Keynesian Phillips curve

- Hence at last, the '*new Keynesian Phillips curve*'

$$\hat{\pi}_t = \kappa \hat{x}_t + \beta \mathbb{E}_t \left\{ \hat{\pi}_{t+1} \right\}, \qquad \kappa \equiv (\sigma + \varphi) \frac{(1-\theta)(1-\theta\beta)}{\theta}$$

  where κ denotes the sensitivity of inflation to current conditions ($\kappa = 0$ if prices rigid, $\theta = 1$ vs. $\kappa = \infty$ if prices flexible, $\theta = 0$)

- Iterating forward

$$\hat{\pi}_t = \kappa \, \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \beta^k \hat{x}_{t+k} \right\}$$

  Expected future booms $\hat{x}_{t+k} > 0$ tend to increase current inflation.
  Expected future busts $\hat{x}_{t+k} < 0$ tend to decrease current inflation.

Dynamic IS curve

- From the log-linear consumption Euler equation

$$r_t = \rho + \sigma \mathbb{E}_t\{\Delta \hat{y}_{t+1}\}$$

- Define the '*natural rate of interest*'

$$r_t^n \equiv \rho + \sigma \mathbb{E}_t\{\Delta \hat{y}_{t+1}^n\}$$

- Therefore

$$r_t - r_t^n = \sigma \mathbb{E}_t\{\Delta \hat{x}_{t+1}\}$$

- This can be written as the '*dynamic IS curve*'

$$\hat{x}_t = -\frac{1}{\sigma}\left(r_t - r_t^n\right) + \mathbb{E}_t\{\hat{x}_{t+1}\}$$

‘Canonical three equation NK model’

(1) New Keynesian Phillips curve

$$\hat{\pi}_t = \kappa \hat{x}_t + \beta \mathbb{E}_t \left\{ \hat{\pi}_{t+1} \right\}, \qquad \hat{x}_t \equiv \hat{y}_t - \hat{y}_t^n$$

(2) Dynamic IS curve, in terms of nominal rate i_t

$$\hat{x}_t = -\frac{1}{\sigma} \left(i_t - \mathbb{E}_t\{\hat{\pi}_{t+1}\} - r_t^n \right) + \mathbb{E}_t\{\hat{x}_{t+1}\}$$

(3) Monetary policy rule, for example

$$i_t = \rho + \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t + v_t$$

Plus essentially exogenous terms and shocks

$$\hat{y}_t^n = \psi_{yz} \hat{z}_t, \qquad r_t^n = \rho + \sigma \psi_{yz} \mathbb{E}_t\{\Delta \hat{z}_{t+1}\}, \qquad \text{shocks } \{v_t, z_t\}$$

Condensed version

- New Keynesian Phillips curve

$$\hat{\pi}_t = \kappa \hat{x}_t + \beta \mathbb{E}_t \left\{ \hat{\pi}_{t+1} \right\}$$

- Dynamic IS curve + monetary policy rule

$$\hat{x}_t = -\frac{1}{\sigma} \left(\phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t - \mathbb{E}_t\{\hat{\pi}_{t+1}\} - u_t \right) + \mathbb{E}_t\{\hat{x}_{t+1}\}$$

- Single '*composite shock*'

$$u_t \equiv \sigma \psi_{yz} \mathbb{E}_t\{\Delta \hat{z}_{t+1}\} - v_t$$

- Two endogenous variables $\hat{\pi}_t, \hat{x}_t$, exogenous composite shock u_t

- Solve the model using the method of undetermined coefficients

Method of undetermined coefficients

- Guess solutions are linear in exogenous composite shock u_t

  $$\hat{x}_t = \psi_{xu}\, u_t$$

  and

  $$\hat{\pi}_t = \psi_{\pi u}\, u_t$$

  for some $\psi_{xu}, \psi_{\pi u}$ to be determined

- For simplicity, suppose u_t is AR(1) with persistence $\rho_u \in (0, 1)$

Method of undetermined coefficients

- From new Keynesian Phillips curve

$$\psi_{\pi u} = \kappa \psi_{xu} + \beta \psi_{\pi u} \rho_u$$

- Hence

$$\psi_{\pi u} = \frac{\kappa}{1 - \beta \rho_u} \psi_{xu} \tag{3}$$

- From the dynamic IS curve

$$\psi_{xu} = -\frac{1}{\sigma} \left(\phi_\pi \psi_{\pi u} + \phi_x \psi_{xu} - \psi_{\pi u} \rho_u - 1 \right) + \psi_{xu} \rho_u \tag{4}$$

- Plug (3) into (4) and solve for ψ_{xu}, then recover $\psi_{\pi u}$ from (3)

Solution

- This gives

$$\psi_{xu} = (1-\beta\rho_u)\Lambda_u$$

$$\psi_{\pi u} = \kappa\Lambda_u$$

where

$$\Lambda_u \equiv \frac{1}{(1-\beta\rho_u)(\sigma(1-\rho_u)+\phi_x)+\kappa(\phi_\pi-\rho_u)}$$

- Notice $\psi_{xu}, \psi_{\pi u}$ have same sign and sign determined by Λ_u, depends on magnitudes of policy coefficients ϕ_π, ϕ_x

- A sufficient condition for $\Lambda_u > 0$ is $\phi_\pi > \rho_u$ (e.g., $\phi_\pi > 1$)

Recover other variables

- Nominal interest rate

$$i_t = \rho + \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t + v_t$$

- Real interest rate

$$r_t = i_t - \mathbb{E}_t \{\hat{\pi}_{t+1}\}$$

- Output

$$\hat{y}_t = \hat{x}_t + \hat{y}_t^n$$

- Employment

$$\hat{l}_t = \hat{y}_t - \hat{z}_t$$

Monetary policy shock

- For simplicity, set $\hat{z}_t = 0$ so $u_t = -v_t$ and $\rho_u = \rho_v$ etc. Assume sufficient reactivity $\phi_\pi > \rho_v \Rightarrow \Lambda_v > 0$

- Inflation falls on impact (in response to contractionary shock)

$$\frac{\partial \hat{\pi}_t}{\partial v_t} = \frac{\partial \hat{\pi}_t}{\partial u_t} \frac{\partial u_t}{\partial v_t} = \psi_{\pi u}(-1) = -\kappa \Lambda_v < 0$$

- Output and output gap fall on impact

$$\frac{\partial \hat{y}_t}{\partial v_t} = \frac{\partial \hat{x}_t}{\partial v_t} = \frac{\partial \hat{x}_t}{\partial u_t} \frac{\partial u_t}{\partial v_t} = \psi_{xu}(-1) = -(1 - \beta \rho_v)\Lambda_v < 0$$

- Employment falls on impact

$$\frac{\partial \hat{l}_t}{\partial v_t} = \frac{\partial \hat{y}_t}{\partial v_t} < 0$$

Monetary policy shock

- Nominal interest rate is ambiguous

$$\frac{\partial i_t}{\partial v_t} = \phi_\pi \frac{\partial \hat{\pi}_t}{\partial v_t} + \phi_x \frac{\partial \hat{x}_t}{\partial v_t} + 1$$

Depends on ρ_v, tends to fall if shock sufficiently persistent

- Real interest rate unambiguously increases

$$\frac{\partial r_t}{\partial v_t} = (\phi_\pi - \rho_v) \frac{\partial \hat{\pi}_t}{\partial v_t} + \phi_x \frac{\partial \hat{x}_t}{\partial v_t} + 1 > 0$$

Example

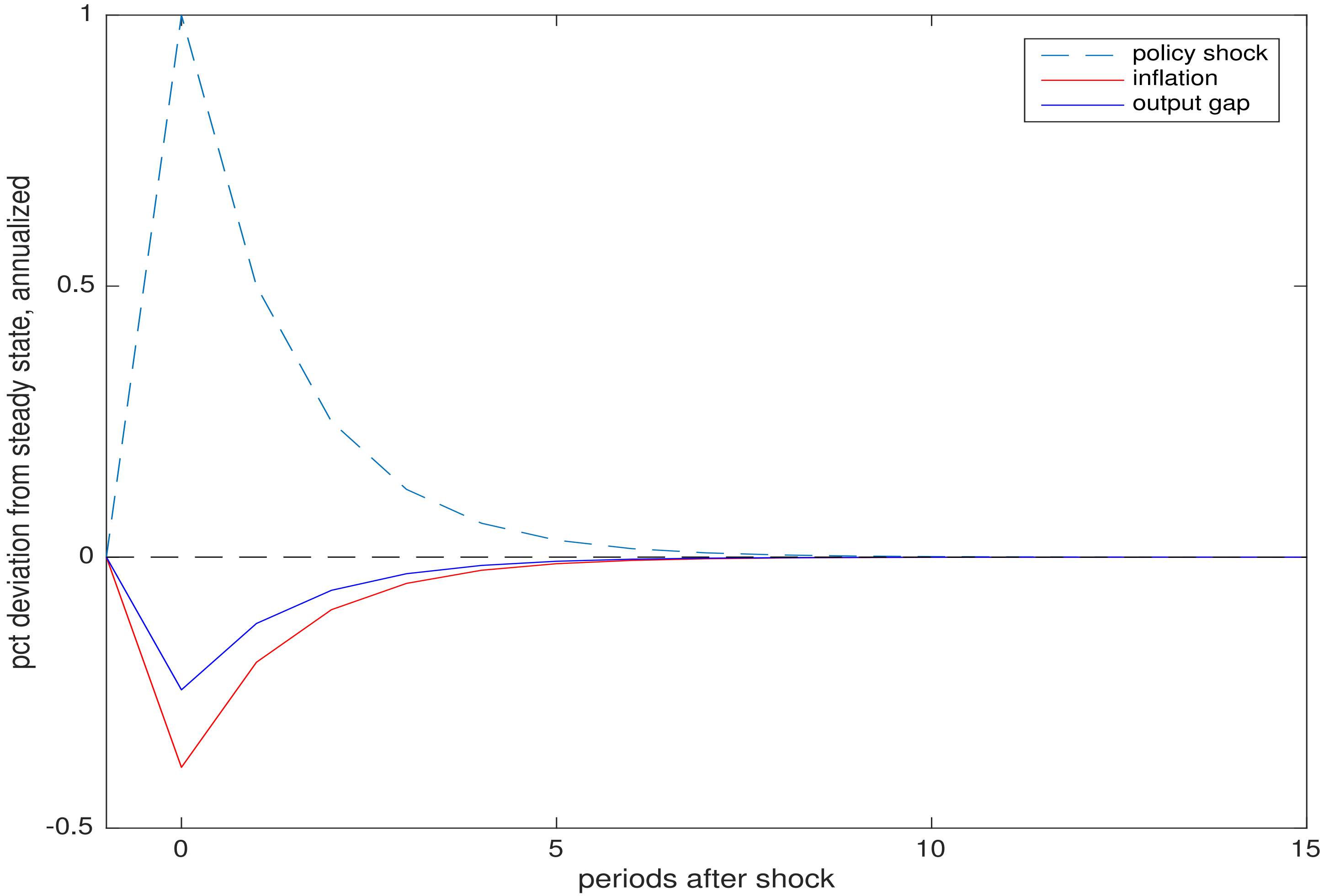
- Let $\beta = 0.99$ quarterly, $\sigma = 1$ and $\kappa = 0.2$
- Suppose monetary policy rule with $\phi_\pi = 1.5$ and $\phi_x = 0.125$
- Suppose monetary policy shock AR(1) with $\rho_v = 0.5$ quarterly
- Equilibrium coefficients work out to

$$\psi_{\pi v} \equiv \frac{\partial \hat{\pi}_t}{\partial v_t} = -\kappa \Lambda_v = -0.39$$

$$\psi_{xv} \equiv \frac{\partial \hat{x}_t}{\partial v_t} = -(1 - \beta \rho_v)\Lambda_v = -0.98$$

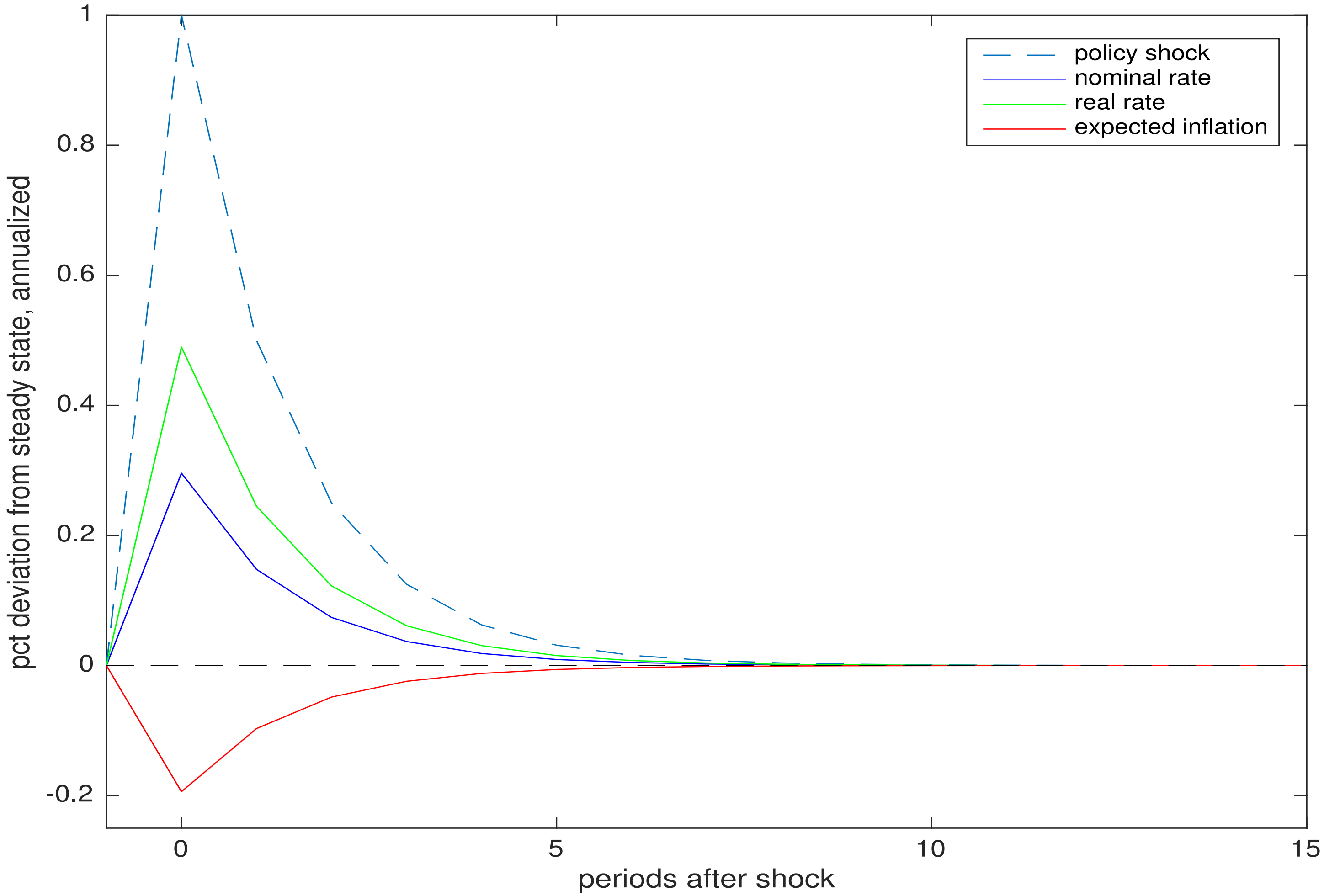
- Note: interest rates, inflation etc expressed at annual rates ($\times 4$)

Monetary policy shock: example



Policy shock $v_t = \rho_v^t$ and implied paths of inflation $\pi_t = \psi_{\pi v} v_t$ and output gap $\hat{x}_t = \psi_{xv} v_t$ (percent deviations from steady state, annualized).

Monetary policy shock: example



Implied paths of nominal interest rate i_t, expected inflation $\mathbb{E}_t\{\hat{\pi}_{t+1}\}$, and real interest rate $r_t = i_t - \mathbb{E}_t\{\hat{\pi}_{t+1}\}$ (percent deviations from steady state, annualized).

Money supply

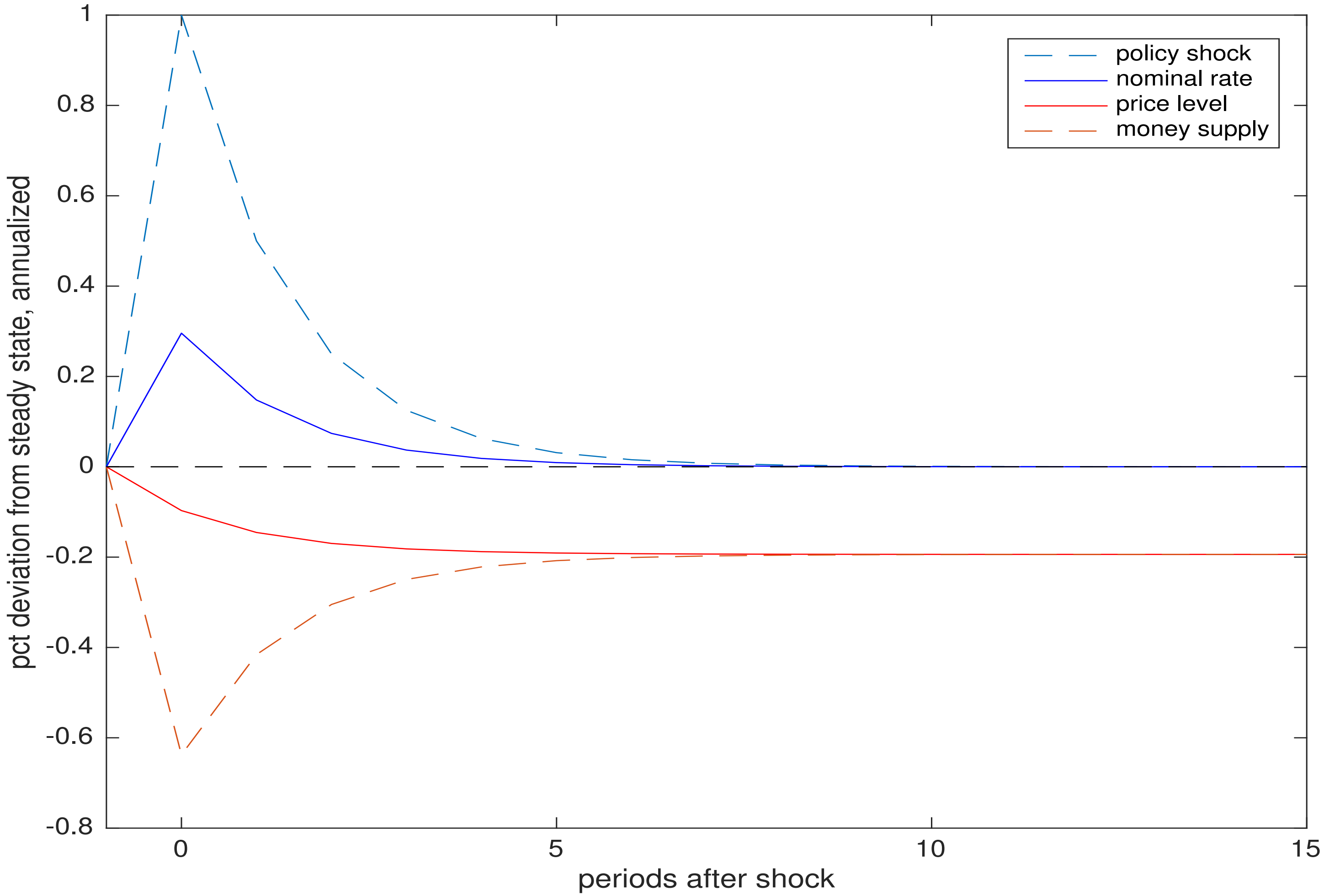
- Suppose money demand is given by

$$\hat{m}_t - \hat{p}_t = \hat{y}_t - \eta i_t$$

- What path for money supply $\hat{m}_t$ is required to implement i_t?

- For this example, set interest semi-elasticity to $\eta = 4$

Monetary policy shock: example



Implied paths of nominal money supply $\hat{m}_t$, price level $\hat{p}_t = \sum_{k=0}^{t} \hat{\pi}_k$ (percent deviations from initial steady state, annualized).