Advanced Macroeconomics

Lecture 10: real business cycles, part two

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1st Semester 2019

This class

- Endogenous labor supply
  - proper RBC model with employment fluctuations
  - numerical examples, building intuition

Representative household

- Period utility over consumption c and labor supply l

$$u(c, l)$$

- Strictly increasing in c, strictly decreasing in l

$$u_{c,t} \equiv \frac{\partial u(c_t, l_t)}{\partial c_t} > 0, \qquad u_{l,t} \equiv \frac{\partial u(c_t, l_t)}{\partial l_t} < 0,$$

Representative household

- Maximizes expected intertemporal utility

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right\}$$

subject to sequence of budget constraints, for each date and state

$$c_t + k_{t+1} \leq w_t l_t + (r_t + 1 - \delta) k_t$$

- Initial $k_0 > 0$ and stochastic processes for wage $\{w_t\}$ and rental rate $\{r_t\}$ taken as given

- *Rational expectations*: household understands mapping from exogenous stochastic processes like productivity to endogenous stochastic processes like wage and rental rate

Representative household

- Lagrangian

$$\mathcal{L} = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) + \sum_{t=0}^{\infty} \lambda_t \left[w_t l_t + (r_t + 1 - \delta) k_t - c_t - k_{t+1} \right] \right\}$$

- Some key first order conditions

$$c_t : \quad \beta^t u_{c,t} - \lambda_t = 0$$

$$l_t : \quad \beta^t u_{l,t} + \lambda_t w_t = 0$$

$$k_{t+1} : \quad -\lambda_t + \mathbb{E}_t \left\{ \lambda_{t+1} \left[r_{t+1} + 1 - \delta \right] \right\} = 0$$

$$\lambda_t : \quad w_t l_t + (r_t + 1 - \delta) k_t - c_t - k_{t+1} = 0$$

Representative household

- New intratemporal condition governing optimal labor supply

$$-\frac{u_{l,t}}{u_{c,t}} = w_t$$

(MRS between labor and consumption equated to real wage)

- Usual consumption Euler equation

$$u_{c,t} = \beta \mathbb{E}_t \left\{ u_{c,t+1} \Big[r_{t+1} + 1 - \delta \Big] \right\}$$

and budget constraint

$$c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t$$

Representative firm

- Maximize profits

$$zf(k,l) - rk - wl$$

- At an optimum, marginal products equal factor prices

$$zf_k = r$$

$$zf_l = w$$

- Note by constant returns of $f(k,l)$

$$f(k,l) = f_k k + f_l l$$

hence

$$rk + wl = zf(k,l)$$

Equilibrium

- Hence in equilibrium, reduces to system

$$-\frac{u_{l,t}}{u_{c,t}} = z_t f_{l,t}$$

and

$$u_{c,t} = \beta \mathbb{E}_t \left\{ u_{c,t+1} \left[z_{t+1} f_{k,t+1} + 1 - \delta \right] \right\}$$

and

$$c_t + k_{t+1} = z_t f(k_t, l_t) + (1 - \delta) k_t$$

- These equilibrium conditions coincide with the optimality conditions of a planner facing the same environment

- Maps exogenous stochastic process $\{z_t\}$ into endogenous stochastic processes $\{c_t, l_t, k_t\}$ and hence $\{w_t, r_t\}$

Standard parameterization

- Separable isoelastic utility function

$$u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\varphi}}{1+\varphi}, \qquad \sigma, \varphi > 0$$

(as we will see, φ controls elasticity of labor supply)

- Production function

$$f(k,l) = k^{\alpha} l^{1-\alpha}, \qquad 0 < \alpha < 1$$

Standard parameterization

- With these functional forms, system becomes

$$l_t^{\varphi} c_t^{\sigma} = (1-\alpha) z_t \Big(\frac{k_t}{l_t}\Big)^{\alpha}$$

and

$$1 = \mathbb{E}_t \left\{ \beta \Big(\frac{c_{t+1}}{c_t}\Big)^{-\sigma} \left(\alpha z_{t+1} \Big(\frac{k_{t+1}}{l_{t+1}}\Big)^{\alpha-1} + 1 - \delta \right) \right\}$$

and

$$c_t + k_{t+1} = z_t k_t^{\alpha} l_t^{1-\alpha} + (1-\delta) k_t$$

Elasticity of labor supply

- Consider labor supply condition

$$l_t^{\varphi} c_t^{\sigma} = w_t$$

or

$$\log l_t = \frac{1}{\varphi} \log w_t - \frac{\sigma}{\varphi} \log c_t$$

- $1/\varphi$ is the '*Frisch*' or 'λ-constant' elasticity of labor supply

- Sensitivity of labor supply to small change in real wage that leaves marginal utility of consumption unchanged

"Non-stochastic steady state"

- Shut down shocks, set $z_t = \bar{z}$

- Capital/labor ratio $\bar{k}/\bar{l}$ solves

$$1 = \beta\left[\alpha\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha-1} + 1 - \delta\right]$$

- Consumption/labor pinned down by resource constraint

$$\frac{\bar{c}}{\bar{l}} = \bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha} - \delta\frac{\bar{k}}{\bar{l}}$$

- Labor $\bar{l}$ then pinned down by labor market clearing

$$\bar{l}^{\varphi+\sigma}\left(\frac{\bar{c}}{\bar{l}}\right)^{\sigma} = (1-\alpha)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}$$

- System can then be log-linearized around $\bar{c}, \bar{l}, \bar{k}$ and solved using method of undetermined coefficients

Method of undetermined coefficients

- Guess solutions linear in *state variables* $\hat{k}_t$ and $\hat{z}_t$

- For the endogenous state variable, capital

$$\hat{k}_{t+1} = \psi_{kk}\hat{k}_t + \psi_{kz}\hat{z}_t$$

- For the *control variables*, consumption and labor

$$\hat{c}_t = \psi_{ck}\hat{k}_t + \psi_{cz}\hat{z}_t$$

$$\hat{l}_t = \psi_{lk}\hat{k}_t + \psi_{lz}\hat{z}_t$$

- In short, we need to determine *six* coefficients

$$\psi_{kk},\ \psi_{ck},\ \psi_{lk}\ ,\ \psi_{kz},\ \psi_{cz}\ \psi_{lz}$$

(we will see how to use Dynare to automate calculations like this)

Inelastic labor supply: recap

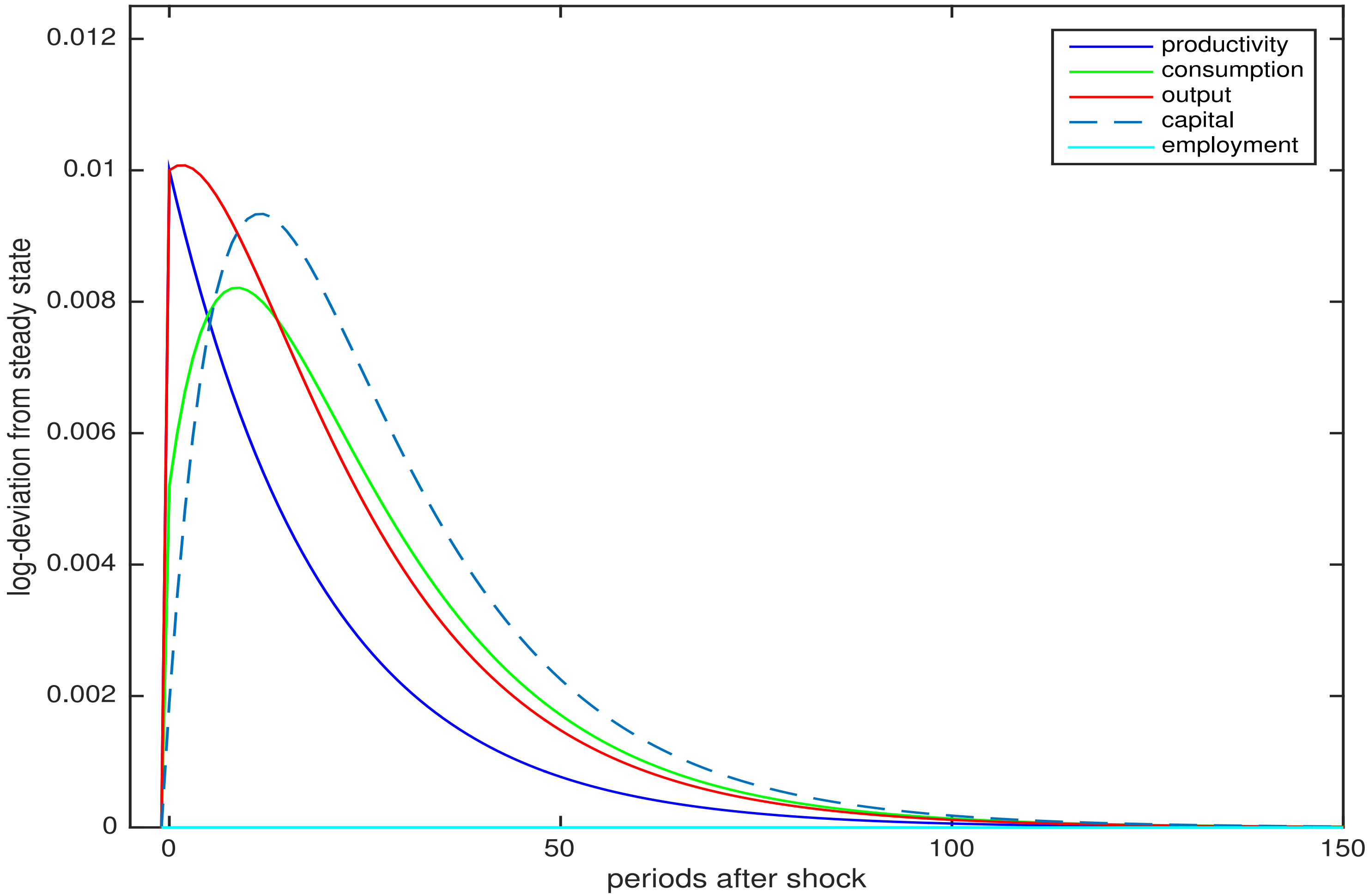
- Let $\varphi \to \infty$ (Frisch elasticity $1/\varphi \to 0$)

- Let $\sigma = \bar{z} = 1$ and $\alpha = 0.3$, $\beta = \phi = 0.95$ and $\delta = 0.05$

- Gives coefficients

$$\begin{pmatrix} \psi_{kk} & \psi_{kz} \\ \psi_{ck} & \psi_{cz} \\ \psi_{lk} & \psi_{lz} \end{pmatrix} = \begin{pmatrix} 0.89 & 0.19 \\ 0.56 & 0.52 \\ 0 & 0 \end{pmatrix}$$

Inelastic labor supply: recap



Capital $\hat{k}_{t+1} = \psi_{kk}\hat{k}_t + \psi_{kz}\hat{z}_t$, consumption $\hat{c}_t = \psi_{ck}\hat{k}_t + \psi_{cz}\hat{z}_t$ and output $\hat{y}_t = \hat{z}_t + \alpha\hat{k}_t$ given $\hat{z}_t = \phi^t\varepsilon_0$ with $\varepsilon_0 = 0.01$ (i.e., +1% productivity shock).

Elastic labor supply

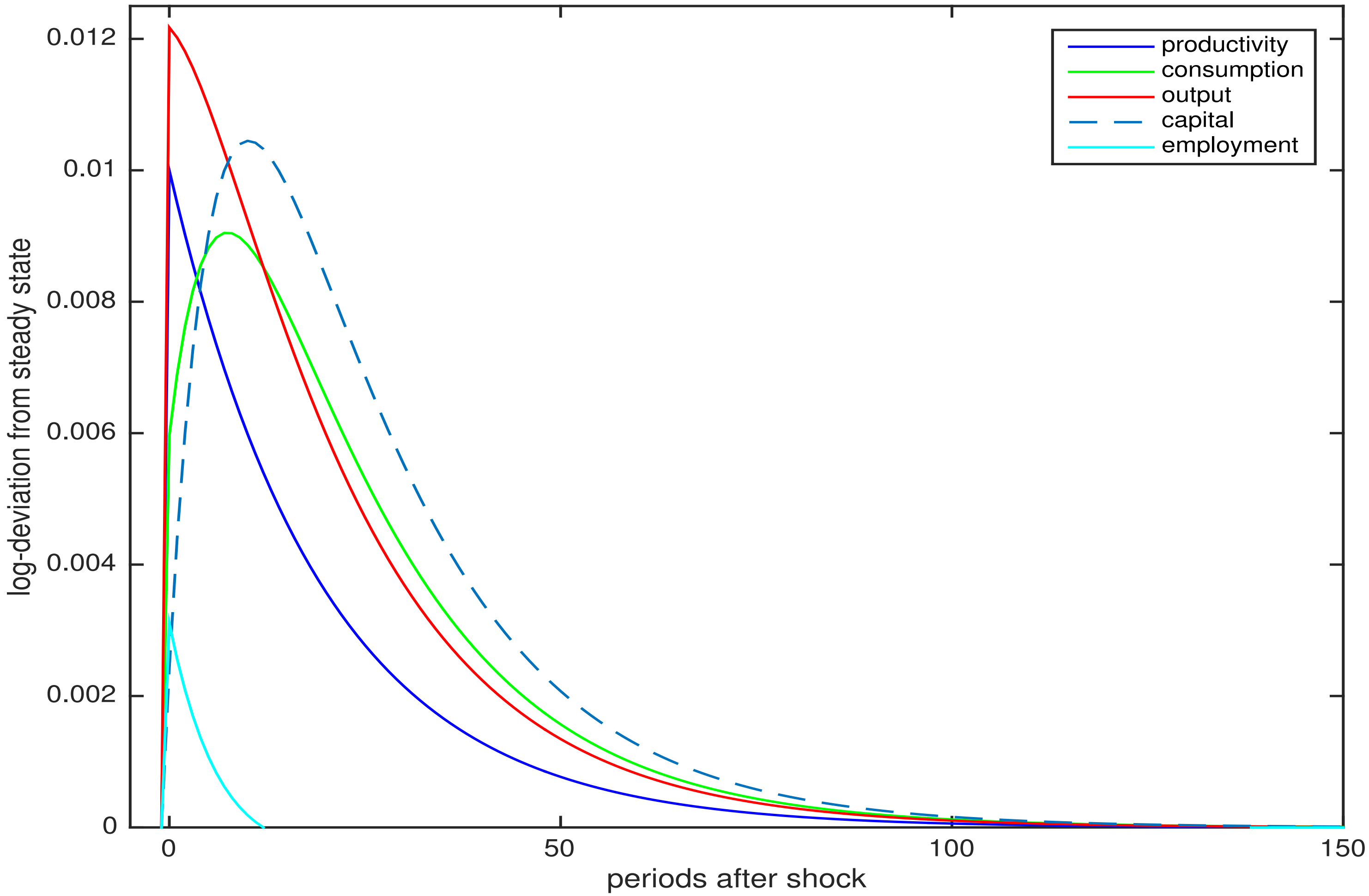
- Now let $\varphi = 1$ (Frisch elasticity $1/\varphi = 1$)

- Keep $\sigma = \bar{z} = 1$ and $\alpha = 0.3$, $\beta = \phi = 0.95$ and $\delta = 0.05$

- Now gives coefficients

$$\begin{pmatrix} \psi_{kk} & \psi_{kz} \\ \psi_{ck} & \psi_{cz} \\ \psi_{lk} & \psi_{lz} \end{pmatrix} = \begin{pmatrix} 0.87 & 0.24 \\ 0.51 & 0.60 \\ -0.16 & 0.31 \end{pmatrix}$$

Elastic labor supply



Capital $\hat{k}_{t+1} = \psi_{kk}\hat{k}_t + \psi_{kz}\hat{z}_t$, consumption $\hat{c}_t = \psi_{ck}\hat{k}_t + \psi_{cz}\hat{z}_t$, employment $\hat{l}_t = \psi_{lk}\hat{k}_t + \psi_{lz}\hat{z}_t$, output $\hat{y}_t = \hat{z}_t + \alpha\hat{k}_t + (1-\alpha)\hat{l}_t$ given $\hat{z}_t = \phi^t\varepsilon_0$, $\varepsilon_0 = 0.01$.

Static special case

- Suppose capital stock is fixed at $\bar{k}$ and does not depreciate, $\delta = 0$

- Implies that c_t, l_t are static functions of z_t

- They simultaneously solve labor market clearing condition

$$l_t^{\varphi} c_t^{\sigma} = (1-\alpha)\bar{k}^{\alpha} z_t l_t^{-\alpha}$$

and resource constraint

$$c_t = z_t \bar{k}^{\alpha} l_t^{1-\alpha}$$

- Or in log-deviations

$$\varphi \hat{l}_t + \sigma \hat{c}_t = \hat{z}_t - \alpha \hat{l}_t$$

$$\hat{c}_t = \hat{z}_t + (1-\alpha)\hat{l}_t$$

Static special case

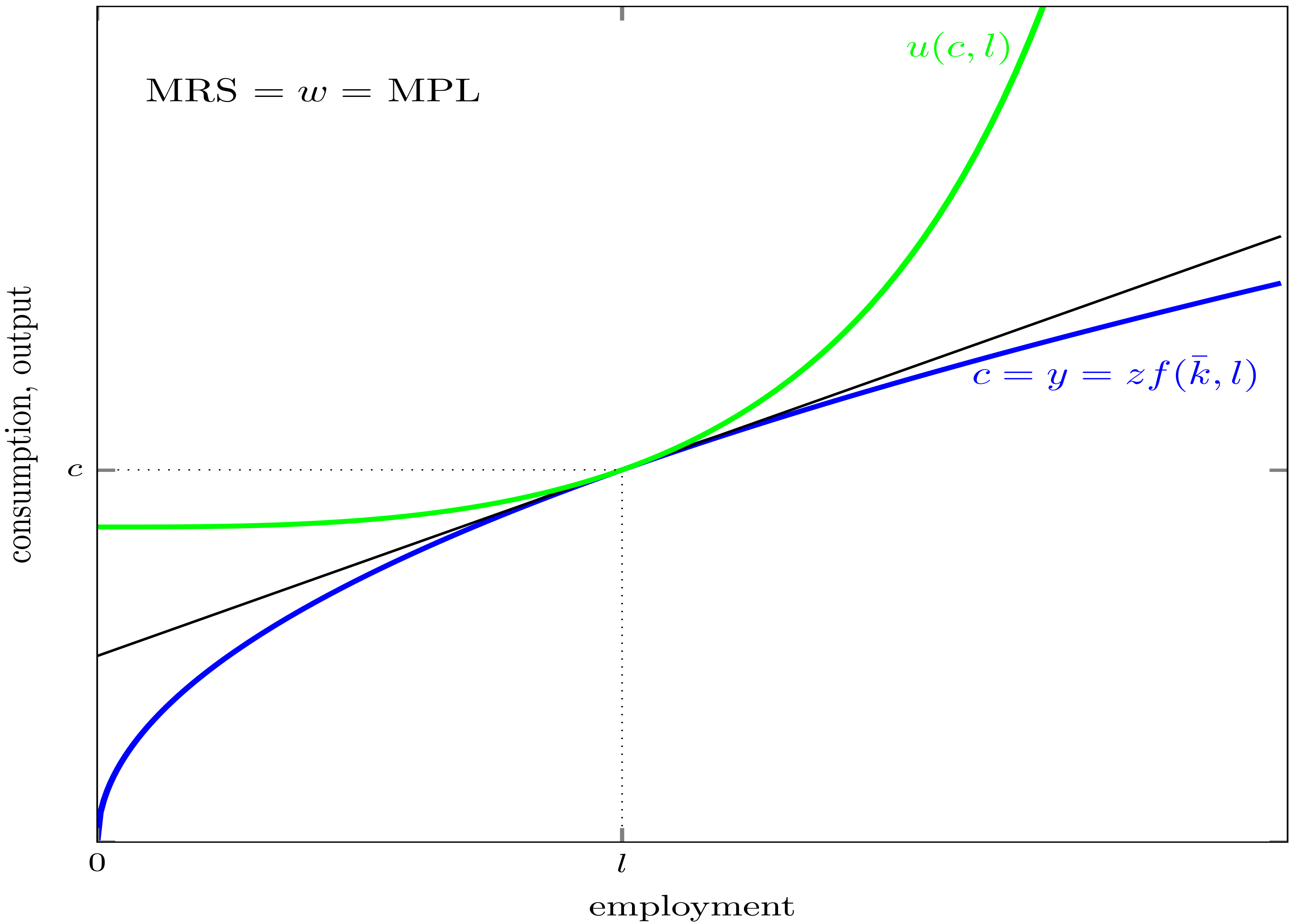
- Solution for this special case

$$\hat{c}_t = \psi_{cz}\,\hat{z}_t, \qquad \psi_{cz} = \frac{1+\varphi}{\alpha + (1-\alpha)\sigma + \varphi} > 0$$

and

$$\hat{l}_t = \psi_{lz}\,\hat{z}_t, \qquad \psi_{lz} = \frac{1-\sigma}{\alpha + (1-\alpha)\sigma + \varphi} \lessgtr 0$$

Static general equilibrium



Next class

- Solving dynamic models in DYNARE
  - getting started
  - stochastic growth example revisited