Advanced Macroeconomics

Lecture 1: introduction and course overview

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1st Semester 2019

This course

An ‘advanced introduction’ to macroeconomics

Core topics in macro — long run growth, business cycles, unemployment, inflation, stabilization policy etc

But greater focus on formal economic models and analytical methods, especially dynamics

Goal is to build intuition and to learn key macro tools, concepts and to make better sense of on-going macro policy debates

A course for anyone who wants to be a *professional economist*, whether in the public sector, private sector, or academia

Course material

- No required text, but a useful supplement
  - David Romer (2012): *Advanced Macroeconomics*. 4th Edition.
- Various journal articles and working papers, posted to the LMS
- Slides for each lecture, posted to the LMS

Assessment

Task	*Due date*	*Weight*
Problem set #1	Wednesday March 27	5%
Problem set #2	Wednesday May 1	5%
Problem set #3	Wednesday May 22	5%
Group presentations	beginning Monday April 29	15%
Final exam	exam block	70%

Group presentations

- In class 30-minute group presentation of research paper
  - list of papers posted to LMS, mix of classics and hot topics
  - 10 groups, 4–5 students per group
  - form group, choose paper (first-come, first-served)
  - schedule meeting with me to discuss preparation
- Presentations in second half of the semester, one per class
- One multiple choice question per presentation on final exam

Tutorials

Tutorial times		
Wednesdays	15:00–16:00	The Spot 2015
Fridays	10:00–11:00	Alan Gilbert 101
Fridays	15:15–16:15	FBE 211 (Theatre 4)

Tutors: Daniel Minutillo and Daniel Tiong.

Tutorials begin next week. Sign up for tutorials asap.

Lecture schedule

- First half: essentially 'frictionless' macro
  - *growth theory and dynamic optimization*, lectures 1–8

    including dynamical systems tools, introduction to Matlab etc
  - *real business cycles*, lectures 9–12
- Problem sets #1 and #2 based on first half of the course

Lecture schedule

- Second half: macroeconomics with frictions, macro policy
  - *monetary economics*, lectures 13–18

    nominal rigidities, new Keynesian models, monetary policy
  - *unemployment*, lectures 19–21

    labor market flows and unemployment fluctuations
  - *financial market frictions*, lectures 22–24

    bank runs, financial crises, endogenous risk
- Problem set #3 based on second half of the course
- Group presentations more in the spirit of second half of course

Rest of this class

Review of discrete-time Solow-Swan growth model

Solow-Swan

- Benchmark model of economic growth, capital accumulation
- Simple setting: no government, closed economy, full employment
- Point of departure vs. earlier Harrod-Domar growth models: aggregate production function with *smooth substitution* between capital and labor
- **Key conclusion:** sustained growth only through productivity growth, not capital accumulation

Setup

- Discrete time $t = 0, 1, 2, \ldots$

- Output Y_t is produced with physical capital K_t and labor L_t according to the *aggregate production function*

$$Y_t = F(K_t, A_t L_t)$$

with *labor-augmenting* productivity A_t

- Physical capital depreciates at rate δ

$$K_{t+1} = (1-\delta)K_t + I_t, \qquad 0 < \delta < 1, \qquad K_0 > 0$$

- For simplicity, productivity A_t and labor L_t grow exogenously

$$A_{t+1} = (1+g)A_t, \qquad 1+g > 0, \qquad A_0 > 0$$

$$L_{t+1} = (1+n)L_t, \qquad 1+n > 0, \qquad L_0 > 0$$

$$Y = F(K, L)$$

- Each input has positive marginal product

$$F_K(K, L) > 0, \qquad F_L(K, L) > 0$$

- Each input suffers from diminishing returns

$$F_{KK}(K, L) < 0, \qquad F_{LL}(K, L) < 0$$

- *Constant returns to scale*, i.e., if both inputs scaled by common factor $c > 0$ then

$$F(cK, cL) = cF(K, L)$$

- Some analysis is simplified by assuming the '*Inada conditions*'

$$F_K(0, L) = F_L(K, 0) = \infty,$$
$$F_K(\infty, L) = F_L(K, \infty) = 0$$

and that both inputs are essential, i.e., $F(0, L) = F(K, 0) = 0$

Savings and investment

- National savings an exogenous fraction of output

$$S_t = sY_t, \qquad 0 < s < 1$$

(key is that savings behavior is exogenous, not that it is constant)

- Hence in a closed economy investment is simply

$$I_t = S_t = sY_t = sF(K_t, A_tL_t)$$

- Putting all this together

$$K_{t+1} = sF(K_t, A_tL_t) + (1-\delta)K_t$$

- *Nonlinear difference equation* in K_t. Starting from initial $K_0 > 0$, determines K_t sequence given A_tL_t sequence and other parameters

Intensive form

- Detrend model by writing in terms of *efficiency units*

$$y \equiv \frac{Y}{AL}, \qquad k \equiv \frac{K}{AL}, \ldots \qquad \text{etc}$$

- Using constant returns to scale

$$y = \frac{Y}{AL} = \frac{F(K, AL)}{AL} = F(\frac{K}{AL}, 1) = F(k, 1) \equiv f(k)$$

- Intensive version of the production function

$$y = f(k), \qquad f'(k) > 0, \qquad f''(k) < 0,$$

and with Inada conditions

$$f'(0) = \infty, \qquad f'(\infty) = 0$$

Capital accumulation

- Then have an *autonomous* difference equation in k_t

$$k_{t+1} = \frac{s}{1+g+n+gn} f(k_t) + \frac{1-\delta}{1+g+n+gn} k_t \equiv \psi(k_t)$$

  starting from initial $k_0 > 0$

- We need to learn how to analyze difference equations like this (and the analogous differential equations in continuous time)

- We will analyze *qualitative* dynamics using a phase diagram

- We will analyze *quantitative* dynamics using local approximations

Steady state

- Steady state capital per effective worker k^* where $k_{t+1} = k_t$, i.e., solves the '*fixed-point*' problem

$$k^* = \psi(k^*)$$

- Equivalently, k^* found where investment per effective worker equals effective depreciation

$$sf(k^*) = (\delta + g + n + gn)k^*$$

- Determines k^* in terms of model parameters $s, \delta, g, n, f(\cdot)$

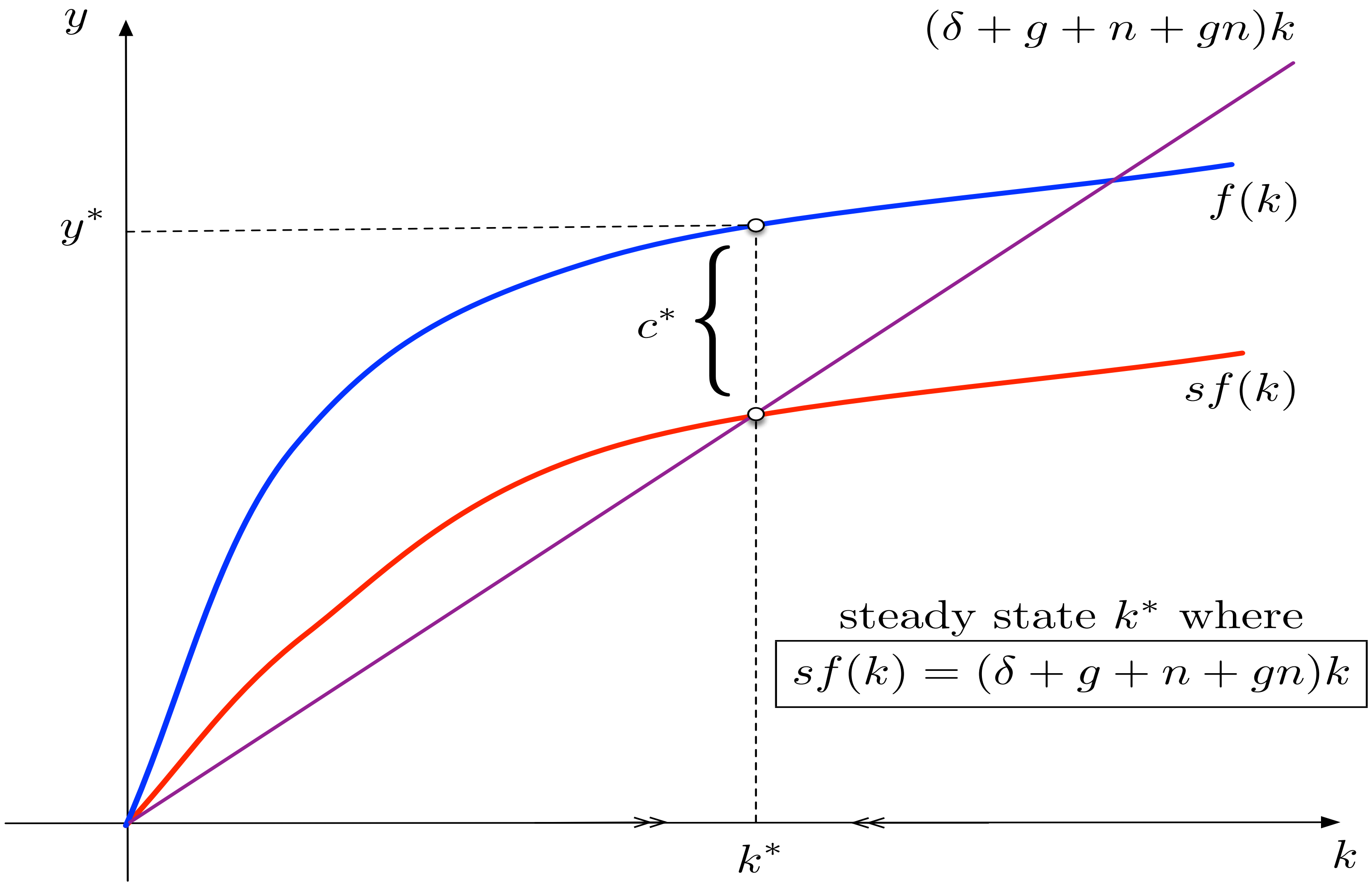
- Then output per effective worker

$$y^* = f(k^*)$$

and consumption per effective worker

$$c^* = (1 - s)f(k^*)$$

Solow diagram



Qualitative dynamics

- Capital per effective worker k_t is *rising* when k_t level is *low*

$$k_{t+1} > k_t \quad \Leftrightarrow \quad sf(k_t) > (\delta + g + n + gn)k_t$$

$$\Leftrightarrow \quad k_t < k^*$$

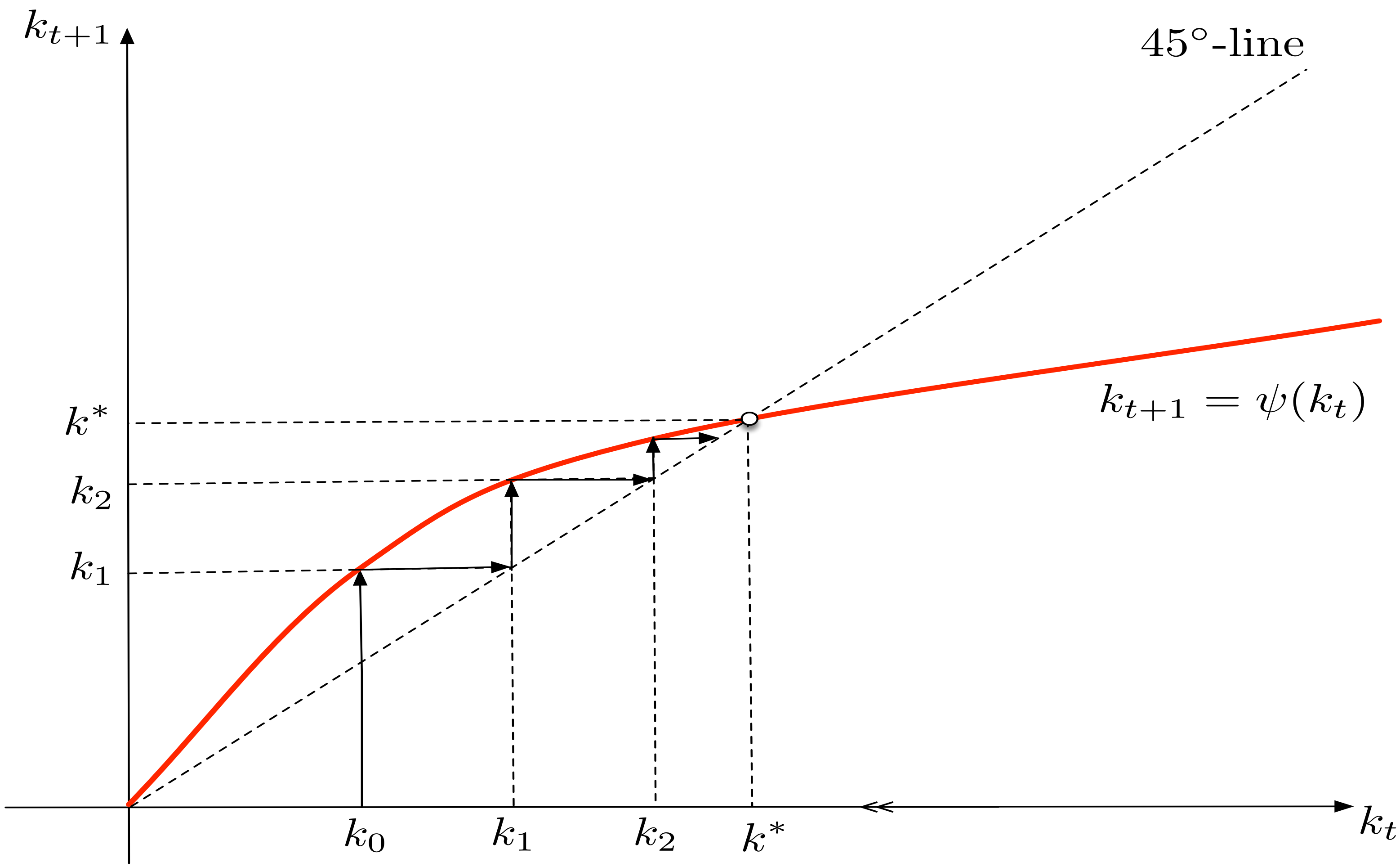
- Capital per effective worker k_t is *falling* when k_t level is *high*

$$k_{t+1} < k_t \quad \Leftrightarrow \quad sf(k_t) < (\delta + g + n + gn)k_t$$

$$\Leftrightarrow \quad k_t > k^*$$

- Converges $k_t \to k^*$, steady state k^* is *stable* (for all $k_0 > 0$). Provides natural notion of 'long run' capital per effective worker

Phase diagram



Linear difference equation

- To understand these stability properties more systematically, let's begin with simple scalar *linear* difference equations, such as

$$x_{t+1} = ax_t + b, \qquad x_0 \text{ given}$$

- Steady state if $a \neq 1$

$$x^* = \frac{b}{1-a}$$

- Can write in *deviations* from steady state

$$(x_{t+1} - x^*) = a(x_t - x^*)$$

Linear difference equation

- Stability properties determined by *magnitude* of coefficient a

- If $a \neq 1$

$$x_t = x^* + a^t(x_0 - x^*), \qquad t = 0, 1, \ldots$$

  If $|a| < 1$ then x_t *converges* to x^* as $t \to \infty$ (quickly if $|a|$ small).
  Monotone convergence if $0 < a < 1$, dampened cycles if $-1 < a < 0$

- If $a = 1$, no steady state and simply

$$x_t = tb + x_0$$

- What if $a = -1$? Steady state at $x^* = b/2$, but *limit-cycle* around x^* except if by chance $x_0 = b/2$ too.

- In short steady state stable if $|a| < 1$ and unstable otherwise.
  In linear system, *local* stability implies *global* stability

Nonlinear difference equation

- Consider scalar nonlinear difference equation

$$x_{t+1} = \psi(x_t), \qquad x_0 \text{ given}$$

- Steady states determined by fixed point problem

$$x^* = \psi(x^*)$$

  May be many, or none

- Stability *local* to a steady state depends on magnitude of $\psi'(x^*)$

Local stability

- To see this, *linearize* around some arbitrary point z

$$x_{t+1} = \psi(x_t) \approx \psi(z) + \psi'(z)(x_t - z)$$

In particular, take $z = x^*$, hence $\psi(z) = x^*$ and

$$x_{t+1} \approx x^* + \psi'(x^*)(x_t - x^*)$$

- In deviations form and treating as exact

$$x_{t+1} - x^* = \psi'(x^*)(x_t - x^*)$$

Linear difference equation with coefficient $a = \psi'(x^*)$

- Hence a given steady state x^* is *locally stable* iff $|\psi'(x^*)| < 1$. But *may not be globally stable.*

Speed of convergence

- Magnitude $\psi'(x^*) = a$ governs speed of convergence. Suppose

$$x_t = x^* + a^t(x_0 - x^*), \qquad 0 < a < 1$$

- How long does it take to close half the $x_0 - x^*$ gap?

$$t = \frac{\log(1/2)}{\log(a)}$$

(i.e., the '*half-life*' of the geometric decay in $x_0 - x^*$)

- Quick convergence when a is low (close to zero), i.e., short half-life

- Slow convergence when a is high (close to one), i.e., long half-life

- In Solow-Swan model, $a = \psi'(k^*)$ determined by parameters $s, \delta, g, n, \ldots$ For what parameter values is a low? high?

Next class

- Solow-Swan model in *continuous time*
  - makes for simpler calculations
  - greater transparency in calibration
- Implications and applications
  - balanced growth path
  - long-run effects of changes in savings rate
  - golden rule
  - speed of convergence
  - examples