Unbundling Labor

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This paper

Provide a new understanding of how changes in

within-occupation wage inequality

can be due to changes in

technology

This paper

1. **Data** - Two new facts

   A. Within occupation residual wage inequality - CPS

   $\uparrow$ High skill occupations , $\downarrow$ Low skill occupations

   B. Similarity of occupations in terms of their skill inputs - O*NET

   $\uparrow$ High skill occupations , $\downarrow$ Low skill occupations

2. **Theory** - Understand **A.** via a comparative static informed by **B.**

   - Extend model of Rosen (1983), Heckman Scheinkman (1987)
   - Endogenize **B.** as appropriate technology choice (Caselli Coleman, 2006)

3. **Extension** - Show that **B.** rationalizes other new facts

   - Declining *experience premium* in low skill occupations
   - Declining *overtime premium / part-time penalty* in low skill occupations
   - Increasing *occupation switching* in low skill occupations

Fact A. - Within occupation wage inequality

Workers in low (high) skill occupations are now paid more (less) similarly

Approach

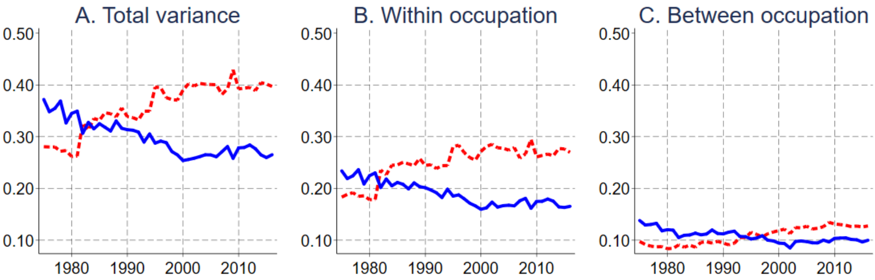
- Split 3 digit occupations into Low skill and High skill
  - Rank by fraction with college education, split by employment
  - Re-classify each year
- Residual wages
  - Residuals from regression of CPS annual earnings $\log y_{it}$ on observables

$$\left[Year_t, NAICS1_{it}, Ed_{it}, Race_{it}, Sex_{it}, FirmSize_{it}, Exp_{it}, Exp_{it}^2, Hours_{it}\right]$$

- Decomposition

$$\underbrace{\mathbb{V}_t\left[e_{ijt}\right]}_{\text{A. Total variance}} = \underbrace{\sum_j \omega_{jt}\mathbb{V}_t\left[e_{ijt}\middle|j\right]}_{\text{B. Within occupation}} + \underbrace{\sum_j \omega_{jt}\Big(\mathbb{E}_t\left[e_{ijt}\middle|j\right] - \mathbb{E}_t\left[e_{ijt}\right]\Big)^2}_{\text{C. Between occupation}}$$

Fact A. - Within occupation wage inequality



Variance of residuals. Red = High skill occupations, Blue = Low skill occupations

1. **Level** Within occupation inequality is important
2. **Change** Low skill occupation workers paid more similarly
3. **Decomposition** Driven by decline in within occupation inequality

Robust across {All,Male,Female}×{Fix occupations in 1980,2010}

▸ Details ▸ Robust - 1980 classification ▸ Robust - 2010 classification

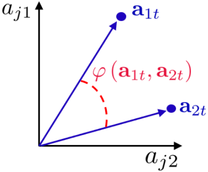
Fact B. - Technology

Low (high) skill occupations have become more similar (more different) in

Approach

1. $J \times K$ matrix of skill measures $\mathbf{A}_t$ from O*NET: 2003-2009, 2010-2018
2. Reduce to $J \times K^*$ matrix of skills $\mathbf{A}_t^*$ (Lise Postel-Vinay, 2020)
3. Distance between occupations (Gathmann Schönberg, 2010)

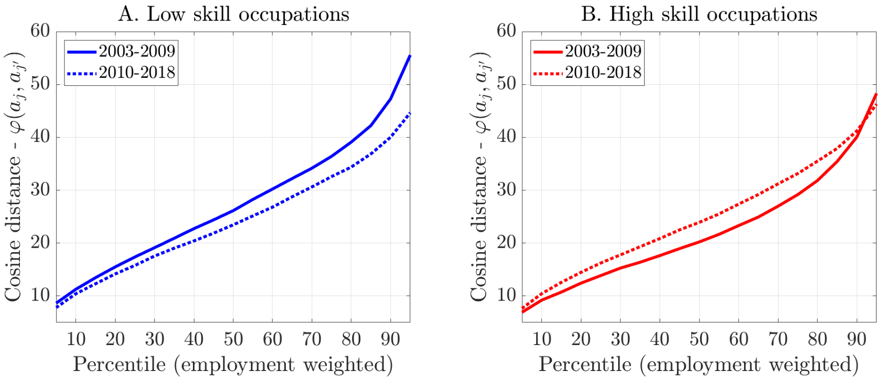
$$\varphi\left(\mathbf{a}_{1t}^*, \mathbf{a}_{2t}^*\right) = \cos^{-1}\left(\frac{\mathbf{a}_{1t}^{*\prime}\ \mathbf{a}_{2t}^*}{||\mathbf{a}_{1t}^*|| \cdot ||\mathbf{a}_{2t}^*||}\right)$$



4. Compare the distribution of these distances $\varphi_{j,j'}$ over time

Details - Dimension reduction

Fact B. - Technology



1. Low skill occupations - More similar - $\downarrow \varphi$
2. High skill occupations - More different - $\uparrow \varphi$

Low skill occupations: Then vs. now

Differentiated technologies

Similar technologies

How does the relative skill bias of technologies across occupations determine wage inequality within occupations?

Model

- General equilibrium environment
  - Individual skills $l(i) = \Big(l_A(i), l_B(i)\Big)$
  - Two occupations $j \in \{1, 2\}$, with different skill intensities

- Competitive equilibrium wages

$$w_j(i) = \omega_{jA} l_A(i) + \omega_{jB} l_B(i) \quad \rightarrow \quad var\Big(\log w_j(i) \Big| j\Big)$$

- Within occupation inequality determined by two forces
  1. Distribution of skills conditional on selection
  2. Gradient of occupation skill prices $\big\{\omega_{jA},\, \omega_{jB}\big\}$

Environment

- Workers $i \in [0,1]$ endowed with two skills $k \in \{A, B\}$

$$l(i) = \Big(l_A(i), l_B(i)\Big) \quad , \quad \Big(l_A(i), l_B(i)\Big) \sim H\Big(l_A, l_B\Big)$$

- Final good

$$U\Big(Y_1, Y_2\Big)$$

- Task / Occupation j technology: $\alpha_1 = (1-\alpha_2) > 0.5$

$$Y_j = F_j\Big(L_{jA}, L_{jB}\Big) = Z_j\Big[\,\alpha_j L_{jA}^{\sigma} \quad + \; (1-\alpha_j)\, L_{jB}^{\sigma}\Big]^{\frac{1}{\sigma}} \quad , \quad \sigma < 1$$

$$L_{jA} = \int l_A(i)\phi_j(i)\, di \;,\; L_{jB} = \int l_B(i)\phi_j(i)\, di \;,\; \phi_j(i) \in \{0,1\}$$

BUNDLED - *Worker i must allocate* $\Big(l_A(i), l_B(i)\Big)$ *to the same task*

Mandelbrot (1962), Rosen (1983), Heckman Scheinkman (1987)

Efficient allocation

$$\max_{\phi_1(i)\in\{0,1\}} U\Big(F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B})\Big)$$

subject to

Let ω_{jk} be the shadow price of L_{jk}

$$L_{1A} = \int \phi_1(i)\, l_A(i)\, di \quad \longrightarrow \quad \omega_{1A} = U_1 F_{1A}$$

$$L_{2A} = \int \Big[1 - \phi_1(i)\Big]\, l_A(i)\, di \quad \longrightarrow \quad \omega_{2A} = U_2 F_{2A}$$

$$L_{1B} = \int \phi_1(i)\, l_B(i)\, di \quad \longrightarrow \quad \omega_{1B} = U_1 F_{1B}$$

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Efficient allocation

$$\max_{\phi_{1A}(i)\in\{0,1\},\phi_{1B}(i)\in\{0,1\}} U\Big(F_1(L_{1A}, L_{1B}), F_2(L_{2A}, L_{2B})\Big)$$

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\end{aligned}$$

and person-by-person bundling constraints

$$\phi_{1A}(i) = \phi_{1B}(i) \qquad \text{for all} \quad i \in [0, 1]$$

Feasible allocations

Replace continuum of individual constraints with a single constraint:

$$\text{BUNDLING CONSTRAINT:} \quad L_{1B} \in \left[\underline{B}(L_{1A})\ ,\ \overline{B}(L_{1A})\right]$$

- Given some L_{1A} what is the *minimum* L_{1B} bundled with it?

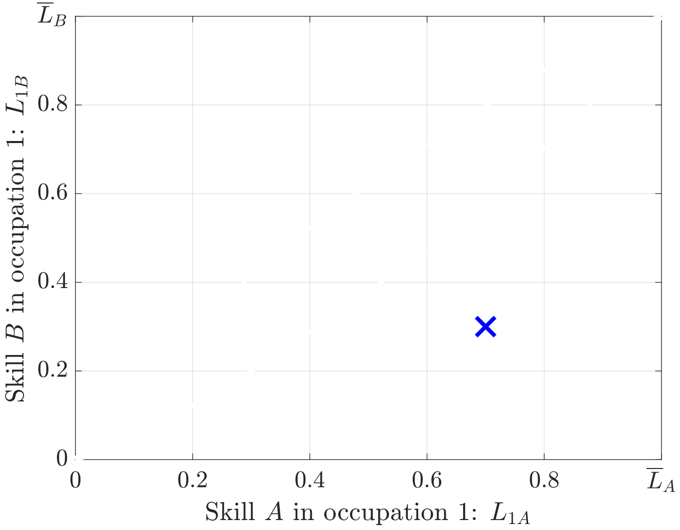
- Construct L_{1A} using workers with highest $l_A(i) \big/ l_B(i)$ first

$$L_{1A} = \int_0^{i^*} l_A(i)\, di \quad , \quad \underline{B}(L_{1A}) = \int_0^{i^*} l_B(i)\, di$$

- Example Let $l_k(i) \sim \textit{Fréchet}(\theta)$ for each skill k

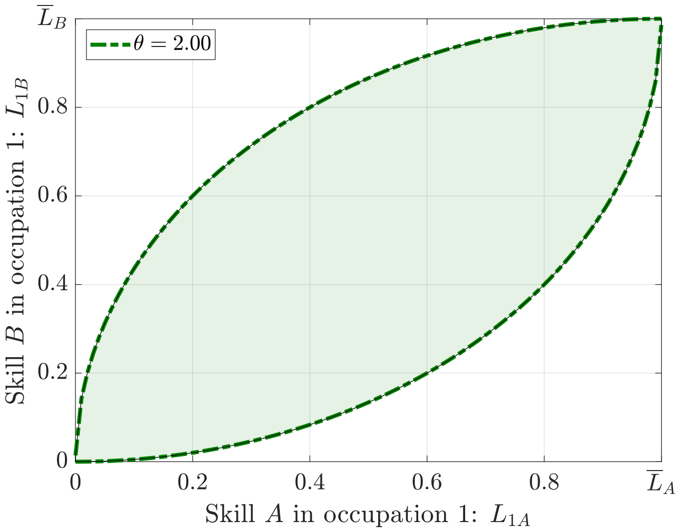
$$\underline{B}\Big(L_{1A}\Big) = \left(1 - \left(1 - \left(\frac{L_{1A}}{\overline{L}_A}\right)^{\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}\right)\overline{L}_B$$

Feasible allocations



Feasible allocations

Feasible allocations must satisfy aggregate *bundling constraints* $L_{1B} \in [\underline{B}(L_{1A}), \overline{B}(L_{1A})]$.
Determined by distribution of skill endowments only. Example: $l_k(i) \sim \textit{Fréchet}(\theta)$.



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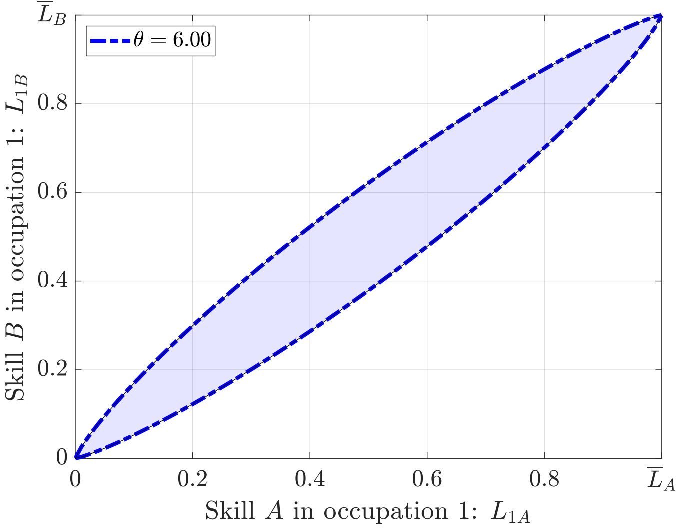

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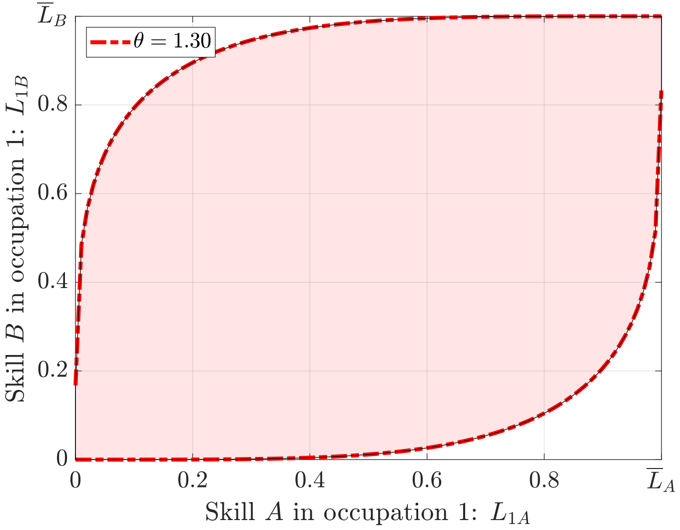

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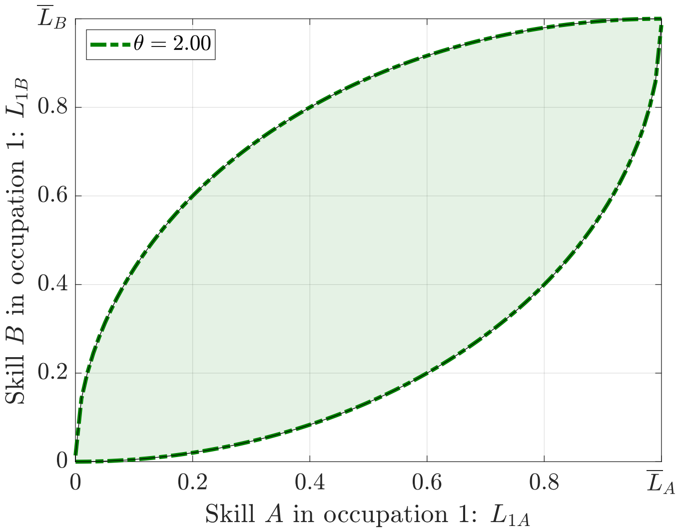


Efficient allocation

$$\max_{L_{1A},L_{1B}} U\Big(F_1\Big(L_{1A},L_{1B}\Big),F_2\Big(\overline{L}_A-L_{1A},\overline{L}_B-L_{1B}\Big)\Big)$$

subject to

$$\underbrace{L_{1B}\geq \underline{B}(L_{1A})}_{\text{Multiplier: } \underline{\beta}}$$

First order conditions

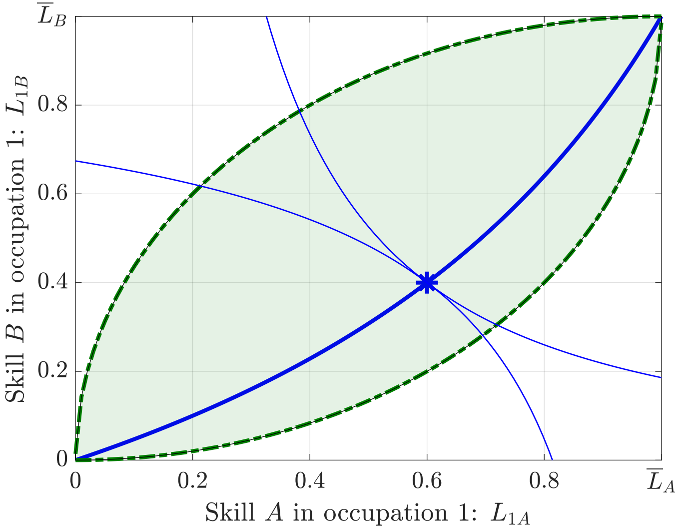
$$\begin{aligned} L_{1A}: \quad & \omega_{1A} & = \quad & \omega_{2A} + \underline{\beta}\,\underline{B}'(L_{1A}) \\ L_{1B}: \quad & \omega_{1B} & = \quad & \omega_{2B} - \underline{\beta} \end{aligned}$$

Results - 1. Same allocation as 'full' problem, 2. Decentralization

Example - Frechet + Cobb-Douglas $\rightarrow$ Closed form comp. stats. for $\underline{\beta}$

Unbundled allocation

'Contract curve' equates marginal rates of technical substitution: $F_{1A}/F_{1B} = F_{2A}/F_{2B}$.
Unbundled allocation equates U_1/U_2 to marginal rate of transformation F_{2k}/F_{1k}.



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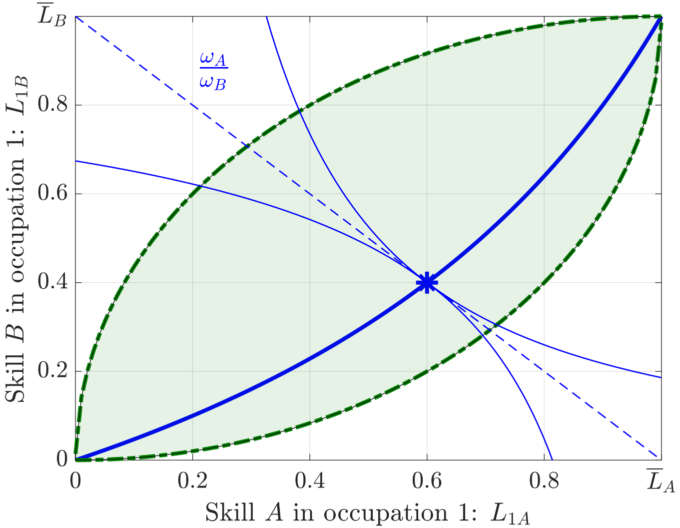
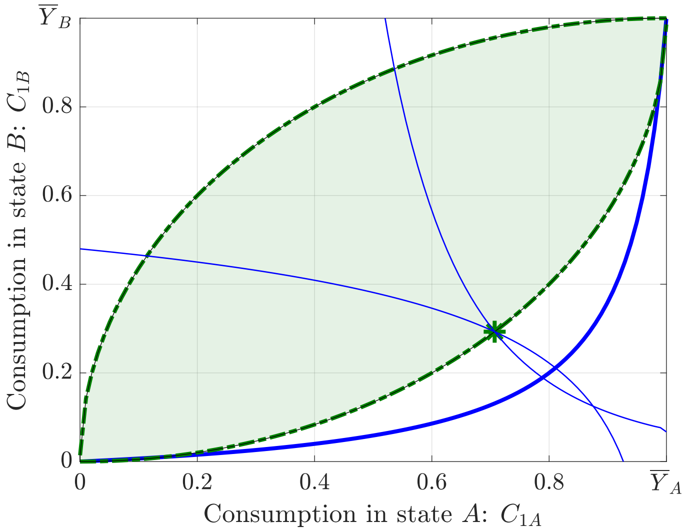


Bundled allocation

Bundling constraint binds. Cannot 'break open' workers to get at underlying skill content.

$$U_1\Big[F_{1A} + \underline{B}'(L_{1A})F_{1B}\Big] = U_2\Big[F_{2A} + \underline{B}'(L_{1A})F_{2B}\Big]$$



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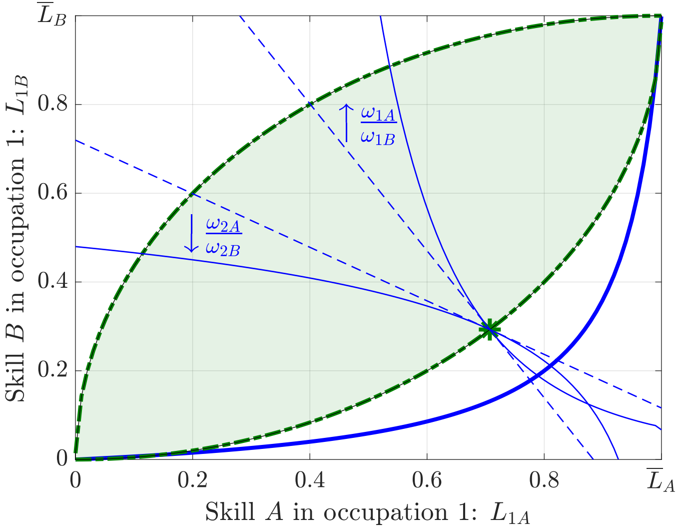
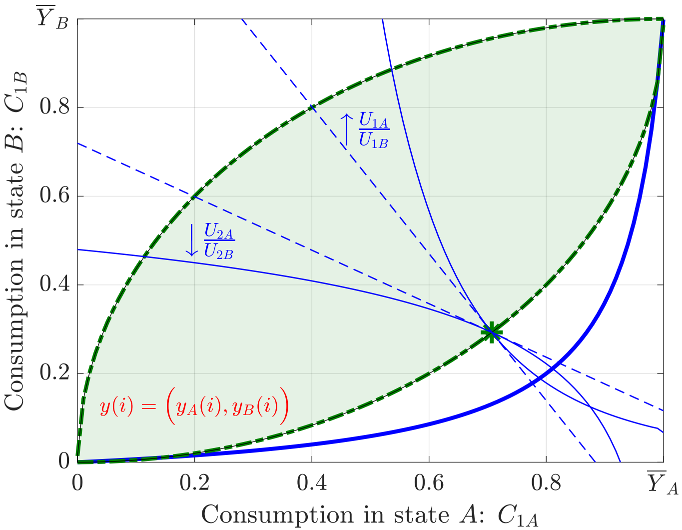


Incomplete markets allocation

Bundling constraint binds. Cannot 'break open' assets to get at underlying arrow securities

$$U_{1A} + \underline{C}'(C_{1A})U_{1B} = U_{2A} + \underline{C}'(C_{1A})U_{2B}$$



Within-occupation skill prices and inequality

1. Wages

$$w_1\left(l_A, l_B\right) = \omega_{1A}\, l_A + \omega_{1B}\, l_B$$

2. Sorting

   - Occupation 1 chosen by individuals with high $\Uparrow l_A \big/ l_B$

3. Inequality

   - Increases as price of primary/secondary skill increases $\Uparrow \omega_{1A} \big/ \omega_{1B}$
   - Decreases as price of primary/secondary skill decreases $\Downarrow \omega_{1A} \big/ \omega_{1B}$

In the paper

- Closed form example under $\left(l_A(i), l_B(i)\right) = \left(e^{\alpha(1-i)}, e^{\alpha i}\right)$
- Log-linear approximation to compute conditional variance
- Decomposes $var\left(\log w(i)|j\right)$ into (i) Endowments, (ii) Prices

▸ Results - Closed form example

Two limiting cases

Illustrate with two nested cases: $\underbrace{\textit{Katz-Murphy}}_{\theta\ \to 1}$ and $\underbrace{\textit{Roy}}_{\alpha_j \to 1}$

1. *'Complete' skill supply* $\Rightarrow$ Always unbundled

$$Y_j = \Big[\, A_{jL} L_L^{\sigma} \;+\; A_{jH} L_H^{\sigma} \Big]^{\frac{1}{\sigma}} \quad , \quad l \in \Big\{ \Big(l_L, 0\Big) \,,\, \Big(0, l_H\Big) \Big\}$$

Law of one price for each skill: ω_A, ω_B

$$var\Big(\log w(i) \,\Big|\, j \Big) = var\Big(\log w(i) \Big)$$

2. *Extreme factor bias* $\Rightarrow$ Always bundled

$$Y_1 = Z_j L_{1A} \quad , \quad L_{1A} = \int l_A(i) \phi_1(i)\, di$$

One positive price for each 'skill': ω_{1A}, ω_{2B}

$$var\Big(\log w(i) \,\Big|\, j \Big) = var\Big(\log l_A(i) \,\Big|\, i < i^* \Big)$$

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$$Y_j = Z_j L_{jA} \quad , \quad L_{jA} = \int l_A(i) \phi_A(i)\, di \quad , \quad l_A(i) = F_1\left(\boldsymbol{x}(i) \right)$$

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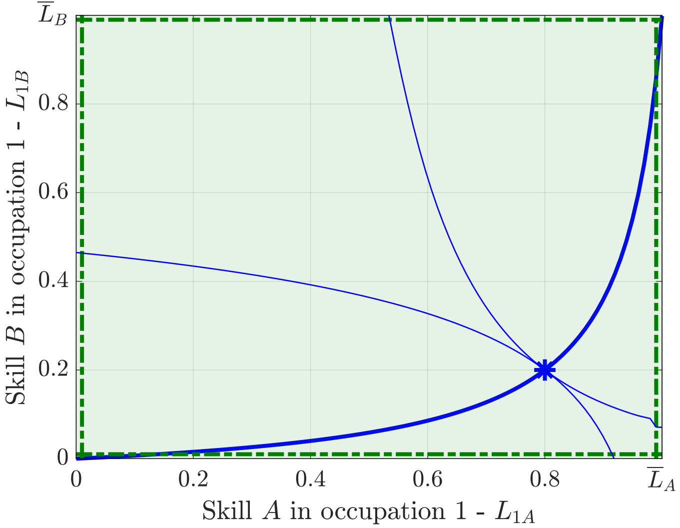
▸ Details - Relationship to the 'Generalized' Roy model

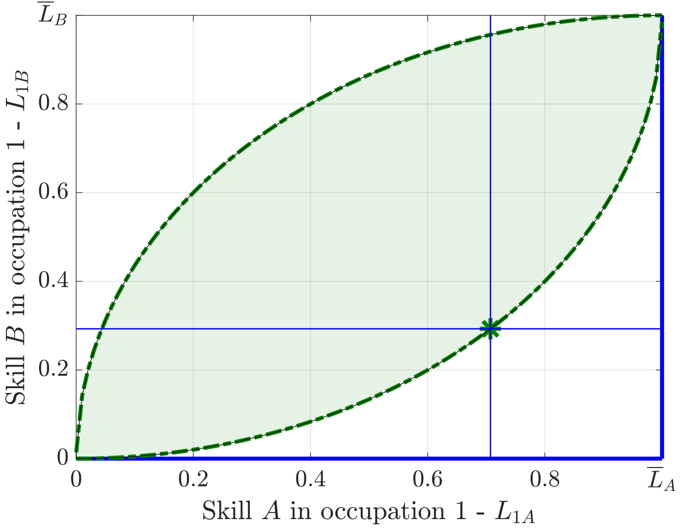
1. Katz-Murphy

Entire set feasible. Equilibrium always unbundled, regardless of technology. Workers not sorted. All workers indifferent. No rents due to comparative advantage. $w_j(i) = \omega_j l_j(i)$



2. Roy

Equilibrium always bundled. Workers sorted by comparative advantage. Skill prices ω_{1A}/ω_{2B} pinned down by relative skills of marginal worker, x^*. $w_j(i) = \omega_j l_j(i)$



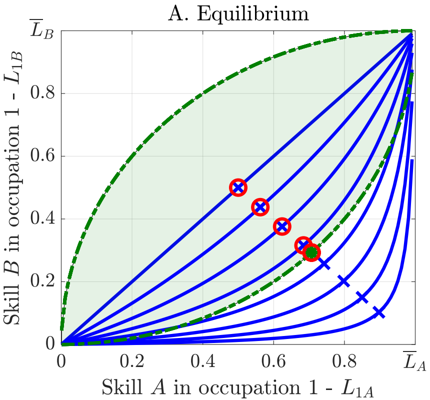
Comparative statics

1. *Symmetric change in factor bias* - $\boldsymbol{\alpha}$
2. *Task-biased change* - Z_1
3. *Skill-biased change* - ψ_A
4. *Task-skill-biased change* - ζ_{1A}

$$
\begin{aligned}
U\Big(Y_1, Y_2\Big) &= \left[\eta Y_1^{\frac{\phi-1}{\phi}} + (1-\eta) Y_2^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \qquad \phi > 1 \\
Y_1 &= Z_1 \Big[\zeta_{1A}\, \psi_A\, \alpha L_{1A}^{\sigma} + (1-\alpha) L_{1B}^{\sigma}\Big]^{\frac{1}{\sigma}} \\
Y_2 &= \Big[\psi_A\, (1-\alpha) L_{2A}^{\sigma} + \alpha L_{2B}^{\sigma}\Big]^{\frac{1}{\sigma}}
\end{aligned}
$$

1. Symmetric change in factor bias

Vary $\alpha_j \in [0.50, 0.85]$. Unbundled: $\omega_{1A} = \omega_{2A}$, $\omega_{1B} = \omega_{2B}$. Bundled: $\omega_{1A} = \omega_{2A} + \underline{B}'(L_{1A})\underline{\beta}$, $\omega_{1B} = \omega_{2B} + \underline{\beta}$. Economy shifts from unbundled equilibrium to bundled equilibrium as $\uparrow \underline{\beta}$



Other parameters: $\sigma = 0.20$, $\phi = 1$, $\theta = 2$, $\overline{L}_A = \overline{L}_B = 1$, $Z_1 = 1$.

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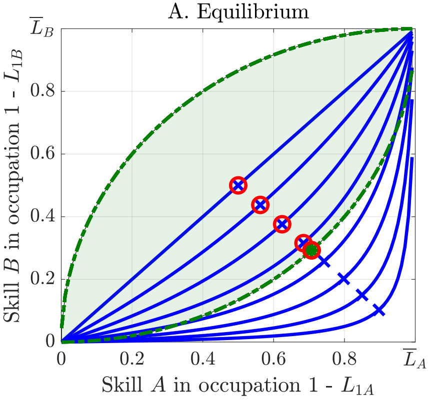

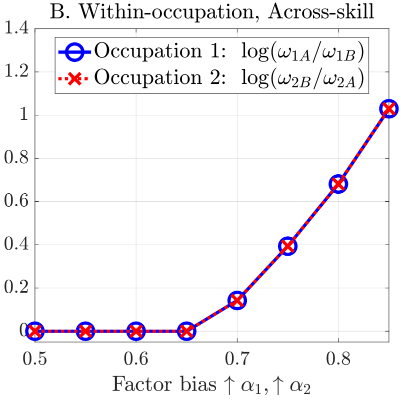

Wage: $w(i) = \omega_{1A} l_A(i) + \omega_{1B} l_B(i)$

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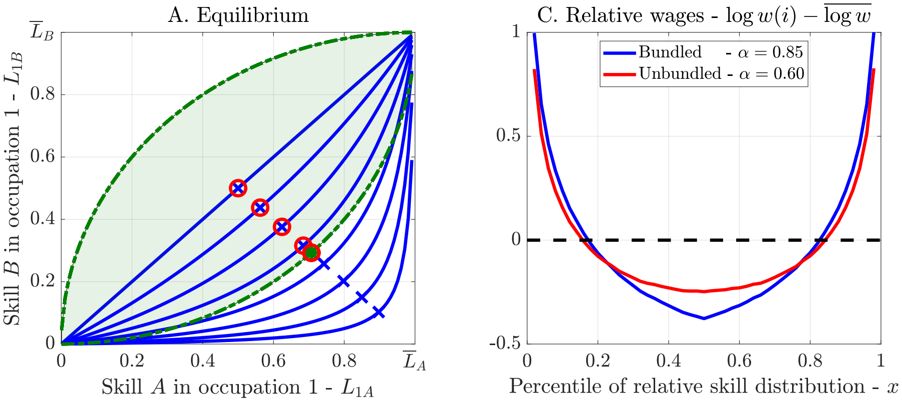

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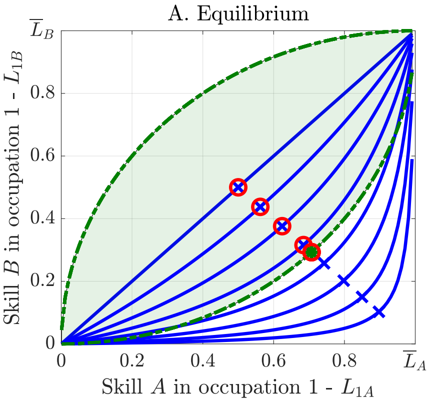
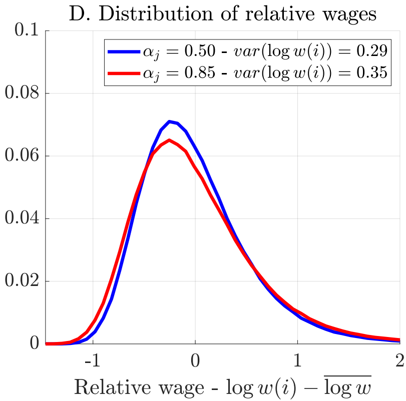




Wage: $w(i) = \omega_{1A} l_A(i) + \omega_{1B} l_B(i)$

Low skill occupations: Then vs. now

$\Uparrow$ *Skill bias* $\rightarrow$ *Bundled / Sorted equilibrium* $\rightarrow$ $\Uparrow$ *Inequality*


$\Downarrow$ *Skill bias* $\rightarrow$ *Unbundled / Unsorted equilibrium* $\rightarrow$ $\Downarrow$ *Inequality*


Under what conditions do these changes in factor intensities emerge endogenously from an expansion in the set of available technologies?

Endogenous technology

Under what conditions do these changes in factor intensities emerge endogenously from an expansion in the set of available technologies?

1. Production function

$$Y_j = \left[\alpha_j \Big(a_{jA} L_{jA}\Big)^{\sigma} + (1-\alpha_j)\Big(a_{jB} L_{jB}\Big)^{\sigma}\right]^{1/\sigma}, \qquad \sigma < 1$$

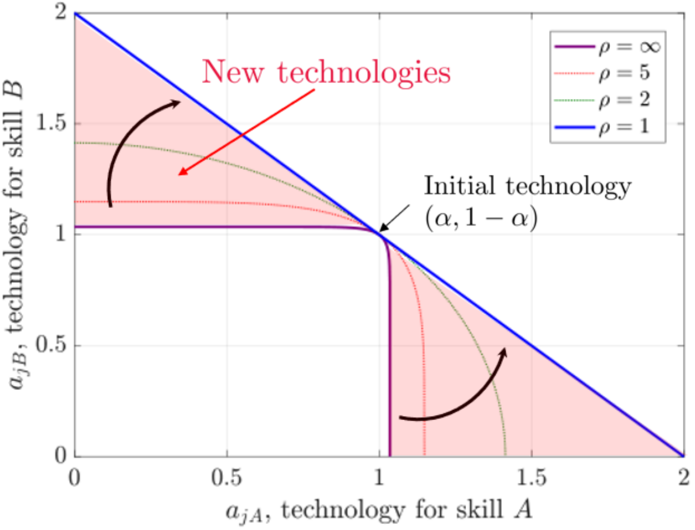
2. Minimize marginal cost subject to *available technologies*

$$\min_{a_{jA}, a_{jB}} \left[\left(\frac{\omega_{jA}}{\alpha_j^{1/\sigma} a_{jA}}\right)^{\frac{\sigma}{\sigma-1}} + \left(\frac{\omega_{jB}}{(1-\alpha_j)^{1/\sigma} a_{jB}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}}$$

$$\text{s.t.} \quad \left[a_{jA}^{\rho} + a_{jB}^{\rho}\right]^{1/\rho} = \overline{A}_j, \qquad \rho > 1$$

Available technologies

Technology frontier $\left[a_{jA}^{\rho} + a_{jB}^{\rho}\right]^{1/\rho} = \overline{A}_j$. As $\rho \searrow 1$ can reach more combinations of a_{jA}, a_{jB} for given $\overline{A}_j$.



Competitive equilibrium

- Skill prices determine technology adoption

$$\omega_{jk} \qquad \Longrightarrow \qquad a^*_{jk}$$

Caselli-Coleman (2006)

- Adopted technology determines sorting and skill premia

$$a^*_{jk} \qquad \Longrightarrow \qquad \underline{\beta} \geq 0 \qquad \Longrightarrow \qquad \omega_{jk}$$

Rosen (1983), Heckman Scheinkman (1987)

Example

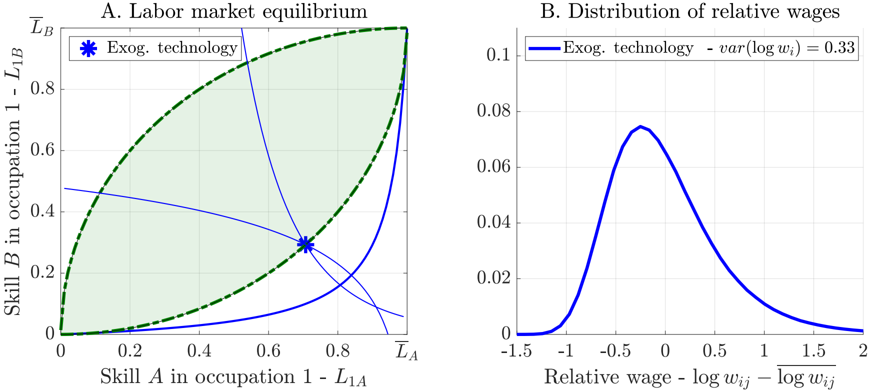
- Symmetric sectors
- Innate skill bias $\alpha_j = 0.8$
- Short-run $\rho = \infty \implies a_{jk} = 1$
- Long-run $\rho = 1$, choose technologies
- Production function CES with e.o.s. σ
- Result

$\sigma > 0$ skills are substitutes $\rightarrow$ *bundling* $\sim$ High skill occupations

$\sigma < 0$ skills are complements $\rightarrow$ *unbundling* $\sim$ Low skill occupations

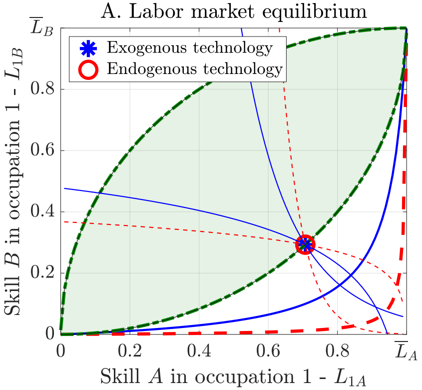
Bundling labor: $\sigma > 0$

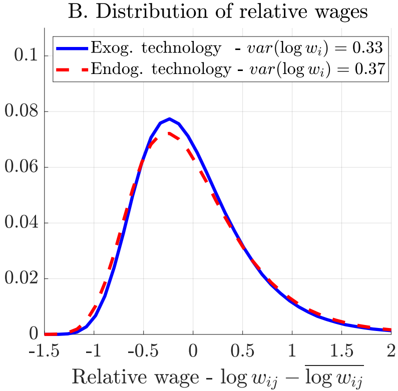
Skills are substitutes, $\sigma > 0$.



Bundling labor: $\sigma > 0$

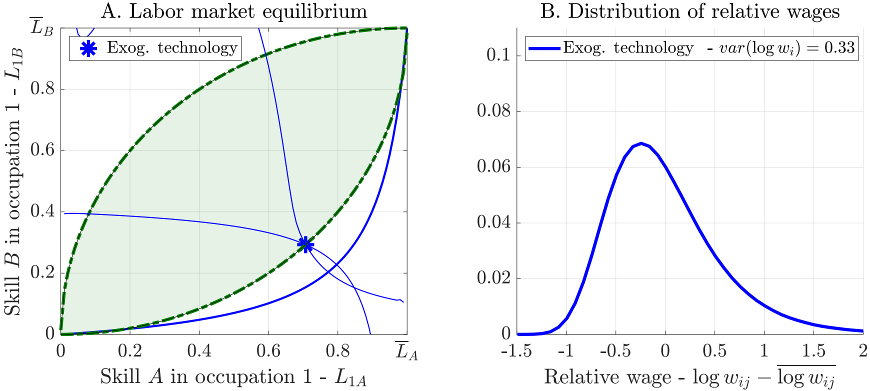
Skills are substitutes, $\sigma > 0$. Choose technology more skill biased. *Endogenously more 'Roy-like'. Bundling constraints tighter. Specialist wages increase. Increasing inequality.*





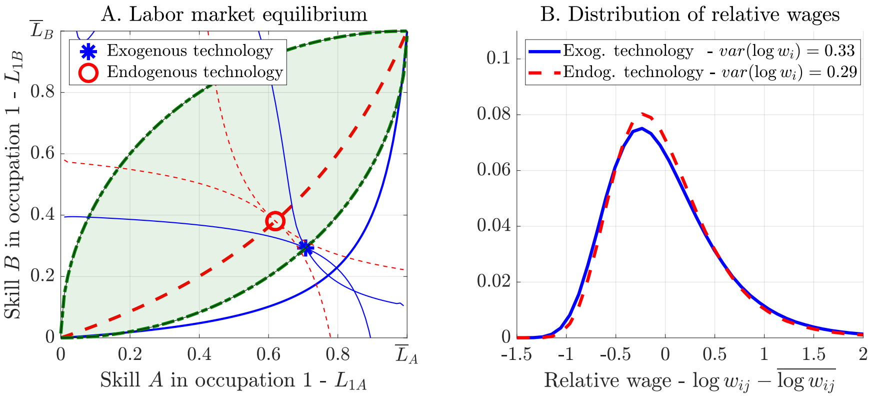
Unbundling labor: $\sigma < 0$

Skills are complements, $\sigma < 0$.



Unbundling labor: $\sigma < 0$

Skills are complements, $\sigma < 0$. Choose technology less skill biased. *Bundling constraints slack. Wage gains for generalists. Wage losses for specialists. Decreasing inequality.*



This paper

1. **Data** - Two new facts

   A. Within occupation residual wage inequality - CPS

   $\uparrow$ High skill occupations , $\downarrow$ Low skill occupations

   B. Similarity of occupations in terms of their skill inputs - OES, O*NET

   $\uparrow$ High skill occupations , $\downarrow$ Low skill occupations

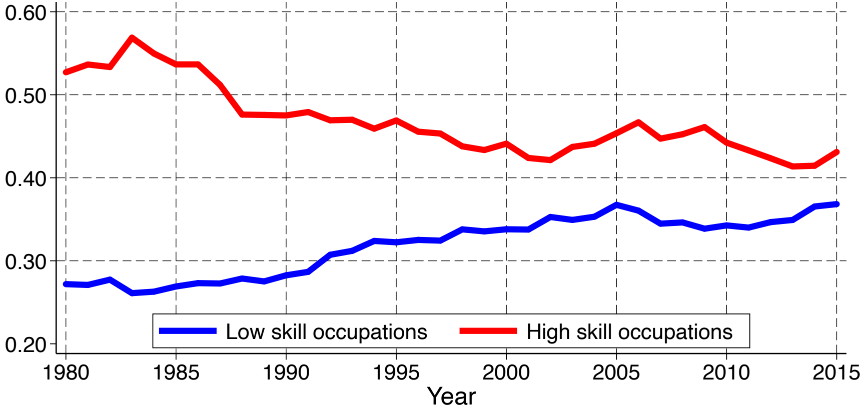
2. **Theory** - Understand **A.** via a comparative static informed by **B.**

   - Extend model of Rosen (1983), Heckman Scheinkman (1987)
   - Endogenize **B.** as appropriate technology choice (Caselli Coleman, 2006)
   - Add participation decision $(l_1, l_2) = (\psi, \psi x)$. Show efficiency properties.

3. **Extension** - Show that **B.** rationalizes other new facts

   - Increasing *occupation switching* in low skill occupations
   - Declining *experience premium* in low skill occupations
   - Declining *overtime premium / part-time penalty* in low skill occupations

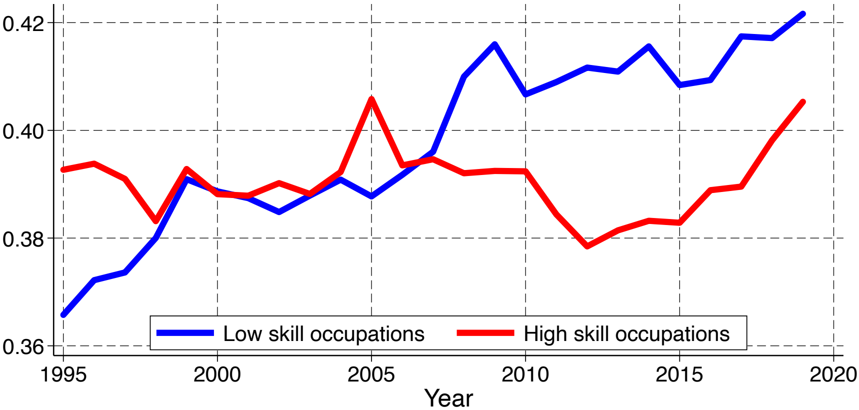
1. Occupation switching



Fraction of **male** workers experiencing $\left\{E_{March}, \ldots, U_m, \ldots, E_{March'}\right\}$

that swap **1-digit** occupations across $\left\{E_{March}, E_{March'}\right\}$

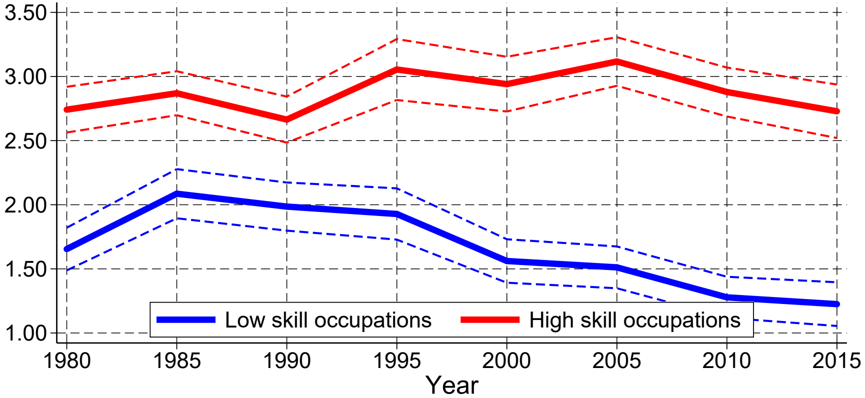
1. Occupation switching



Fraction of **male** workers experiencing $\left\{E_{Month}, E_{Month+1}\right\}$

that swap **1-digit** occupations across $\left\{E_{Month}, E_{Month+1}\right\}$

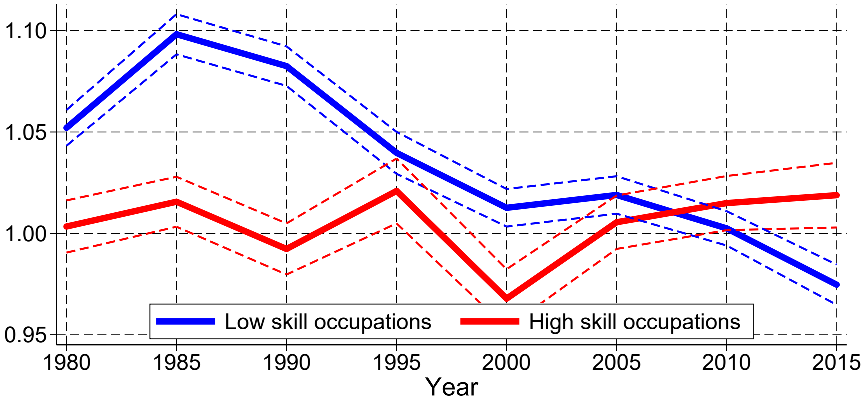
2. Experience premium



One extra year experience associated with 2 to 3 percent higher wage

$$\log Inc_{it} = \alpha + \beta^{\tau}_{Hours} \log Hours_{it} + \beta^{\tau}_{Exp} Exp_{it} + \beta^{\tau}_{Exp^2} Exp^2_{it} + \beta^{\tau}_{Size} Size_{it} \ldots$$
$$+\beta^{\tau}_{X} \left[Year_t, Race_{it}, NAICS1_{it}, Ed_{it}, Sex_{it}\right]$$

3. Hours premium



$(= 1)$: wage independent of hours, (≥ 1): wage increasing in hours

$$\log Inc_{it} = \alpha + \beta^{\tau}_{Hours} \log Hours_{it} + \beta^{\tau}_{Exp} Exp_{it} + \beta^{\tau}_{Exp^2} Exp^2_{it} + \beta^{\tau}_{Size} Size_{it} \ldots$$
$$+ \beta^{\tau}_{X} \left[Year_t, Race_{it}, NAICS1_{it}, Ed_{it}, Sex_{it} \right]$$

Interpreting other facts

1. Increasing *occupation switching* in low skill occupations

- Unbundled equilibrium features indeterminate occupational choice

2. Declining *experience premium* in low skill occupations

- Add learning by doing in the direction of occupation skill bias
  Cavounidis Lang (JPE, 2020)
- Experience premium $\leftrightarrow$ Inframarginal rents
- Unbundling labor reduces gradient of primary / secondary skill prices
- Reduces observed experience premium

3. Declining *overtime premium / part-time penalty* in low skill occupations

- Requires more work to extend the model
- Unbundling labor $\leftrightarrow$ Workers are more ‘substitutable’

Conclusions

- Deviations from law of one price for skills if either
  (i) technologies sufficiently factor biased, or
  (ii) weak pattern of comparative advantage in skills

- Can generate opposite trends in within-occupation wage inequality from technology adoption

- If skills *substitutes*, technology adoption *tightens bundling constraints*
  $\uparrow$ returns to comparative advantage, $\uparrow$ sorting
  $\uparrow$ within-occupation wage inequality
  Consistent with experience of *high skill occupations*

- If skills *complements*, technology adoption *can cause unbundling*
  $\downarrow$ returns to comparative advantage, $\downarrow$ sorting
  $\downarrow$ within-occupation wage inequality
  Consistent with experience of *low skill occupations*

Appendix

Link to Bais, Hombert, Weill (2020)

- Setup - Two agents $j \in \{1,2\}$ consume in two states $k \in \{A, B\}$
- Preferences - Expected utility of consumption

$$F_j\left(C_{jA}, C_{jB}\right) = \pi_A \alpha_j \frac{C_{jA}^{1-\gamma}}{1-\gamma} + \pi_B \left(1-\alpha_j\right) \frac{C_{jB}^{1-\gamma}}{1-\gamma} \quad , \quad \alpha_1 > \frac{1}{2} > \alpha_2$$

- Trees - Physical assets indexed $i \in [0,1]$ have payoffs

$$d(i) = \left(d_A(i), d_B(i)\right) \quad , \quad d_A(i)/d_B(i) \text{ decreasing in } i$$

- Budget constraints - Period-0 and Period 1, State-k

$$\begin{aligned} \int Q(i)\phi_j(i)\,di + q_A a_{jA} + q_B a_{jB} &\leq \phi_j^0 \int Q(i)\,di \\ C_{jk} &= \int \phi_j(i) d_k(i)\,di + a_{jk} \end{aligned}$$

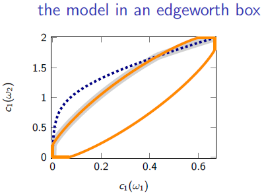
- Incentive compatibility - Only short arrow securities up to $(1-\delta)$ of tree payoffs

$$C_{jk} \geq \delta \int \phi_j(i) d_k(i)\,di \quad , \quad k \in \{A, B\} \quad \text{Slack if } \delta = 0. \text{ No shorts if } \delta = 1$$

- Feasibility - What IC (C_{1A}, C_{2A}) can be supported by a set of trees?

$$C_{1A} = \delta \int_0^{k^*} d_A(i)\,di \rightarrow k^*(C_{1A}) \rightarrow \underline{C}_{1B}\left(C_{1A}\right) \geq \delta \int_0^{k^*(C_{1A})} d_B(i)\,di$$

Link to Bais, Hombert, Weill (2020)



a graphical analysis of the incentive feasible set (IF set)

- area inside the orange curve: IF set with many trees and $\delta < 1$
- dotted-blue curve: Pareto set without IC constraints
- highlighted-grey curve: Pareto set with IC constraints

- Here w/out IC, trees redundant. Trade in Arrow securities. $Q(i) = \sum_k q_k d_k(i)$.
- If IC binds, ratios of marginal utilities not equated: $\omega_{1A}/\omega_{1B} > \omega_{2A}/\omega_{2B}$
- The price of tree i depends on which agent j holds it

$$Q_1(i) = q_A d_A(i) + (q_B - \delta\mu_{1B})\, d_B(i)\,,\; Q_2(i) = (q_A - \delta\mu_{1A})\, d_A(i) + q_B d_B(i)$$

- In equilibrium $\omega_{1A} > \omega_{2A}$ and $\omega_{1B} < \omega_{2B}$, which implies $\omega_{1A} > \omega_{1B}$
- Result - Securities with more extreme pay-offs (specialists) are more expensive
- Result - Price of tree encodes constraint, lower than replicating arrow securities

Competitive equilibrium

$$\begin{aligned}
\Pi_1 &= \max_{L_{1A}, L_{1B}} P_1 F_1\Big(L_{1A}, L_{1B}\Big) - Cost_1\Big(L_{1A}, L_{1B}\Big) \\
Cost_1\Big(L_{1A}, L_{1B}\Big) &= \min_{\widetilde{\phi}_1(i)} \int \widetilde{\phi}_1(i) w_1(l_A, l_B)\, di
\end{aligned}$$

subject to

$$\begin{aligned}
L_{1A} &= \int \widetilde{\phi}_1(i)\, l_A\, di \qquad \longrightarrow \quad \omega_{1A} = P_1 F_{1A} \quad \Big(MC_{1A} = MRPL_{1A}\Big) \\
L_{1B} &= \int \widetilde{\phi}_1(i)\, l_B\, di \qquad \longrightarrow \quad \omega_{1B} = P_1 F_{1B} \quad \Big(MC_{1B} = MRPL_{1B}\Big)
\end{aligned}$$

Labor demand for each type

$$\widetilde{\phi}_1(i) = \begin{cases} 1 & , \quad \text{if} \quad \omega_{1A} l_A(i) + \omega_{1B} l_B(i) > w_1\big(l_A, l_B\big) \\ 0 & , \quad \text{if} \quad \omega_{1A} l_A(i) + \omega_{1B} l_B(i) < w_1\big(l_A, l_B\big) \\ \in (0,1) & , \quad \text{if} \quad \omega_{1A} l_A(i) + \omega_{1B} l_B(i) = w_1\big(l_A, l_B\big) \end{cases}$$

▸ Back - Two allocations

Competitive equilibrium

- Prices per efficiency unit of skill

$$\begin{aligned} w_j\Big(l_A, l_B\Big) &= \omega_{jA} l_A + \omega_{jB} l_B \\ \omega_{jk} &= P_j F_{jk} = U_j F_{jk} \end{aligned}$$

- Worker (l_A, l_B) chooses occupation $j = 1$ only if

$$w_1\Big(l_A, l_B\Big) > w_2\Big(l_A, l_B\Big)$$

- Cutoff worker indifferent

$$\underbrace{\frac{\omega_{1A} - \omega_{2A}}{\omega_{2B} - \omega_{1B}}}_{\text{Benefit of } j=1} = \underbrace{\left(\frac{l_B}{l_A}\right)^*}_{\text{Relative skill in } j=2} = \underline{B'}\Big(L_{1A}\Big)$$

Under $\{\omega_{jk} = U_j F_{jk}\}$, this is the same condition as in the planner's problem

▸ Back - Two allocations

Competitive equilibrium

- *Bundled equilibrium*: Sorting premia are increasing in $\underline{\beta}$

$$\omega_{1A} - \omega_{2A} = \underline{\beta}\,\underline{B}'(L_{1A})$$
$$\omega_{2B} - \omega_{1B} = \underline{\beta}$$

– Inframarginal workers earn rents due to comparative advantage, determined by sorting premia.

– Additional source of within-occupation wage inequality

- *Unbundled equilibrium*: Sorting premia are zero, indeterminate sorting

$$\omega_{1A} - \omega_{2A} = 0$$
$$\omega_{2B} - \omega_{1B} = 0$$

– All workers are marginal. No rents due to comparative advantage.

Back - Two allocations

Generalized Roy model

- Individual-occupation specific output

$$y_j(i) = \exp\Big(\alpha_{jA} l_A(i) + \alpha_{jB} l_B(i)\Big) \quad , \quad Y_j = \int \phi_j(i) y_j(i)\, di$$

- The *only* priced objects are $y_1(i)$, $y_2(i)$ with prices w_1, w_2

$$\log w_j(i) = \log w_j + \alpha_{jA} l_A(i) + \alpha_{jB} l_B(i)$$

- In our case

$$\log w_j(i) \approx \log \overline{w}_j + \widetilde{\omega}_{jA} \widehat{l}_A(i) + \widetilde{\omega}_{jB} \widehat{l}_B(i)$$

1. Technology affects wages directly through the technology coefficients
2. Within occupation inequality effects are silo-ed:
   - Suppose that technology changes in occupation 2
   - All changes in the economy are encoded in the *occupation skill price* w_j, i.e. the occupation fixed effect
   - No change in incumbent *within occupation inequality* in occupation 1

▸ Back - When is the equilibrium bundled?

Wage inequality - Closed form example

- Skills for individuals $i \in [0, 1]$

$$\Big(l_A(i), l_B(i)\Big) = \Big(\gamma e^{\alpha(1-i)}, \gamma e^{\alpha i}\Big) \quad \rightarrow \quad l_B(i)/l_A(i) = e^{\alpha(2i-1)}$$

- Approximate log wage around mean log skills conditional on selection i^*

$$\log w(i,j) = \log \left[\omega_{1A} e^{\log l_A(i)} + \omega_{1B} e^{\log l_B(i)}\right]$$

- Within occupation inequality

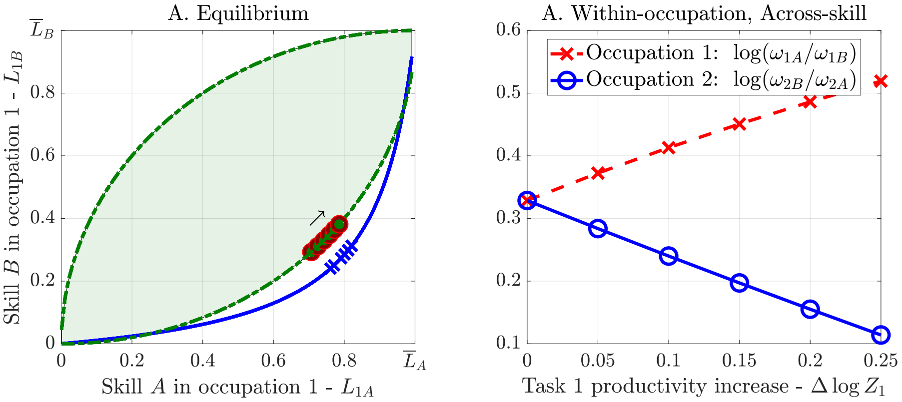
$$var\Big(\log(w(i)) \Big| j^*(i) = 1\Big) = \underbrace{\left(\frac{\left(\frac{\omega_{1A}}{\omega_{1B}}\right) e^{\alpha(1-i^*)} - 1}{\left(\frac{\omega_{1A}}{\omega_{1B}}\right) e^{\alpha(1-i^*)} + 1}\right)}_{\text{Bundling}} \underbrace{\alpha^2 \frac{i^{*\,2}}{12}}_{\text{Roy}}$$

1. **Roy** As $\omega_{1A}/\omega_{1B} \rightarrow \infty$, bundling terms goes to zero
2. **Bundling** With finite ω_{1A}/ω_{1B}, inequality increasing in ratio

Back - Wage inequality

2. Task-Biased Change

Exogenous $\uparrow Z_1$, with $\phi > 1$: $\uparrow Y_1$, $\downarrow Y_2$.
Marginal worker has more Skill B, pushes up ω_{1A}/ω_{1B}. Opposite for task 2.



Other parameters: $\alpha_{1A} = \alpha_{2B} = 0.80$, $\sigma = 0.20$, $\theta = 2$, $\overline{L}_1 = \overline{L}_2 = 1$, $Z_2 = 1$.

Back - Comparative statics

2. Task-Biased Change

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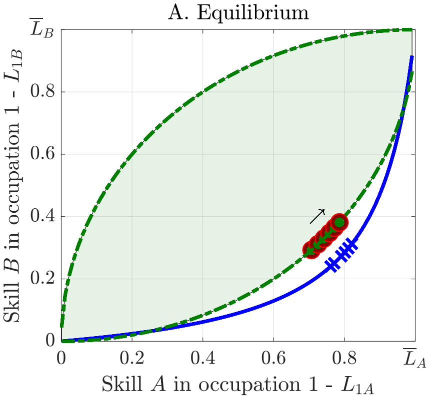

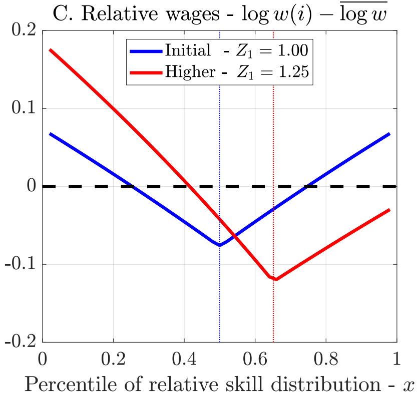

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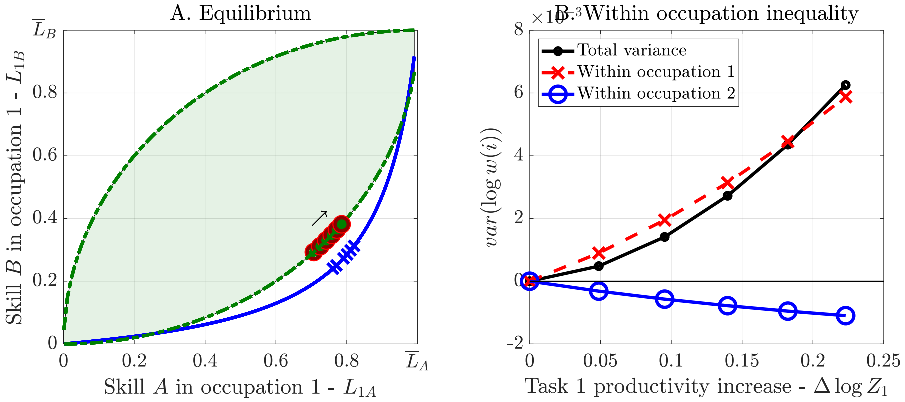

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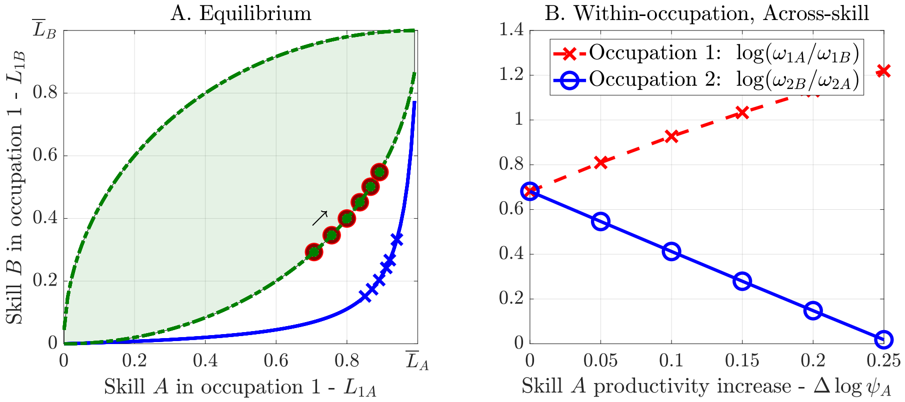
▸ Back - Comparative statics

3. Skill-Biased Change

Exogenous $\uparrow \psi_A$, with $\phi > 1$, $\sigma > 0$: $\uparrow Y_1$, $\downarrow Y_2$.
Marginal worker has more Skill B, pushes up ω_{1A}/ω_{1B}. Opposite for task 2.



Other parameters: $\alpha_{1A} = \alpha_{2B} = 0.80$, $\sigma = 0.20$, $\theta = 2$, $\overline{L}_1 = \overline{L}_2 = 1$, $Z_1 = Z_2 = 1$.

Back - Comparative statics

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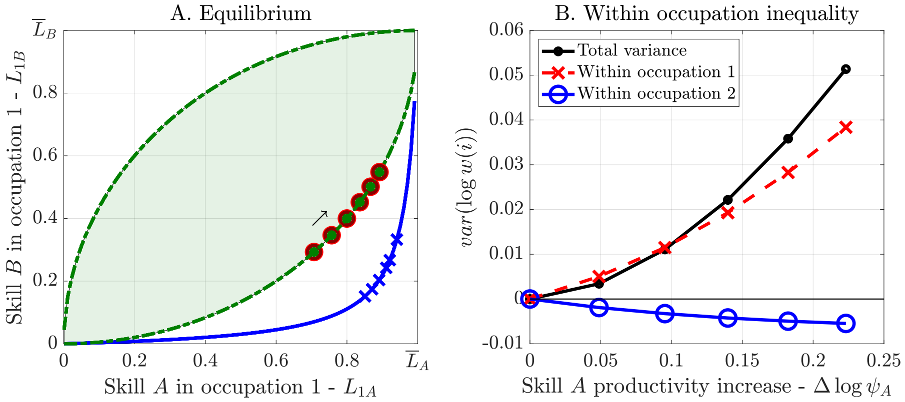

Other parameters: $\alpha_{1A} = \alpha_{2B} = 0.80$, $\sigma = 0.20$, $\theta = 2$, $\overline{L}_1 = \overline{L}_2 = 1$, $Z_1 = Z_2 = 1$.

Back - Comparative statics

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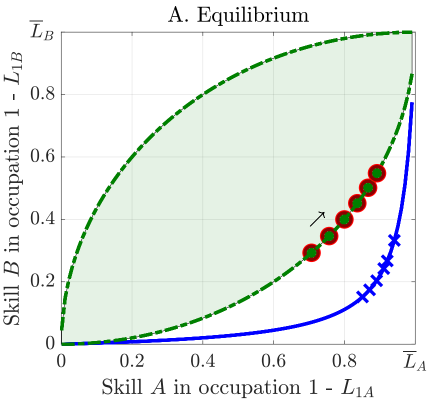

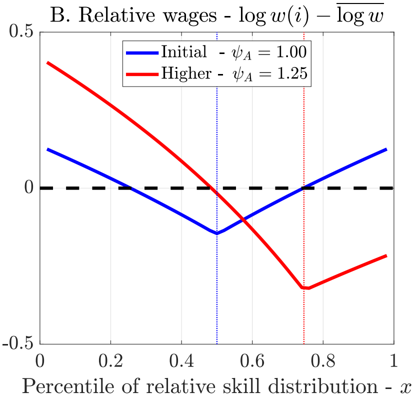

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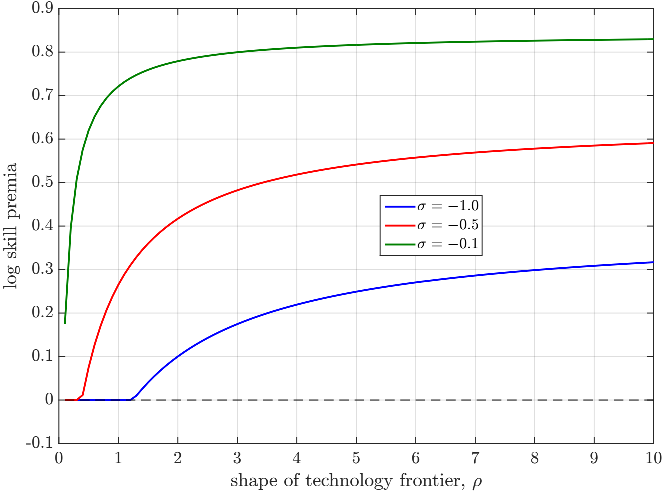
Back - Comparative statics

Unbundling Labor: $\downarrow \rho$, $\sigma < 0$

As ρ falls, technologies become 'more substitutable'. If $\sigma < 0$, firms undo existing skill bias, bundling constraints loosen, skill premia fall, wage gains for generalists. $p_A = \omega_{1A} - \omega_{2A}$



Extensions I

- Absolute vs. comparative advantage

$$\left(l_1, l_2\right) = \left(\psi, \psi x\right) \quad , \quad \left(\psi, x\right) \sim H\left(\psi, x\right)$$

+ fixed utility of being out of the labor market

- Selection on x *margin* (occupation) and on ψ *margin* (participation)

- RESULT: Competitive equilibrium allocation is efficient

- What are the effects of adding a mass of *low-productivity unspecialized workers* ($\downarrow \psi$, $x \approx 1$)?

(sr) wages and allocations for fixed technology

(lr) wages and allocations for endogenous technology

Empirics - Details

- All data based on March CPS 'last year' questions

- Occupation, Industry - Dorn's 1990 harmonized cross-walk
  - Drop military
  - Occupation skill = Fraction of workers with high-school or less
  - Occupations sorted on occupation skill

- Use HPV (RED, 2010)
  - Earnings = Wage income + $(2/3)\times$ Self employment income
  - Annual hours = *Weeks worked last year* $\times$ *Usual hours worked per week*
  - Wage = Earnings / Annual hours
  - Age 25-65, Wage $>$ $0.5\times$ Federal minimum wage, Hours $>$ One month of 8hr days

- Regression controls for residualized wage:
  - Worker education (3 levels), Industry (1 digit), Experience, Experience^2 Race, Log hours,
  - Experience = (age - max(years in school,12)) - 6

Back - Motivating empirics

Empirics - Regressions

1. Workers in low skill occupations getting paid more 'similarly'.

   - Reduced form empirical evidence from the CPS

$$\log Earnings_{i,t} = \gamma_t + \delta^{Occ}_{period} + \boldsymbol{\beta}'_{period}\mathbf{X}_{i,t} + \varepsilon_{i,t}$$

$$\mathbf{X}_{i,t} = \left[Year_t, NAICS1_{it}, Ed_{it}, Race_{it}, Sex_{it}, FirmSize_{it}, Exp_{it}, Exp^2_{it}, Hours_{it}\right]$$

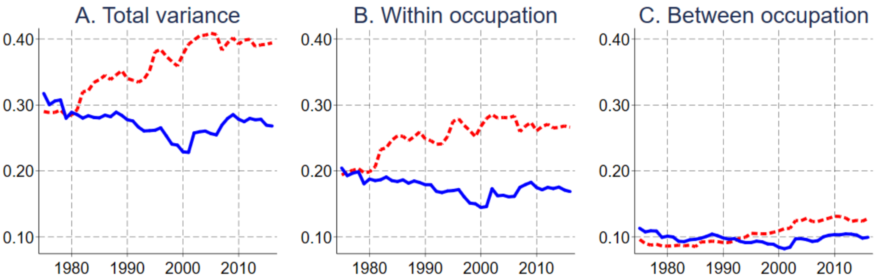
   - Low skill: Decline in $\downarrow \widehat{\beta}_{period}$ for (i) experience, (ii) hours, (iii) large firm
   - High skill: No change

2. Anecdotal evidence from US labor market

   - Goldin Katz (2012) vs. David Weil (2014)
   - Hard to explain declining level of 'attachment' of working age men

Back - Motivating empirics

Data - Wage inequality



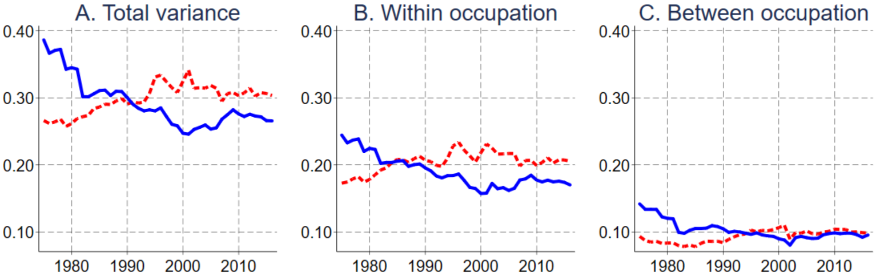
$$\underbrace{\mathbb{V}_t\left[\log \widetilde{y}_{ijt}\right]}_{\text{A. Total variance}} = \underbrace{\sum_j \omega_{jt}\mathbb{V}_{jt}\left[\log \widetilde{y}_{ijt}\right]}_{\text{B. Within occupation}} + \underbrace{\sum_j \omega_{jt}\left(\mathbb{E}_{jt}\left[\log \widetilde{y}_{ijt}\right] - \mathbb{E}_t\left[\log \widetilde{y}_{ijt}\right]\right)^2}_{\text{C. Between occupation}}$$

- Red = High skill occupations, Blue = Low skill occupations
- 3 digit occupations - **Classified in 2010**

$$\mathbf{X}_{i,t} = \left[Year_t, NAICS1_{it}, Ed_{it}, Race_{it}, Sex_{it}, FirmSize_{it}, Exp_{it}, Exp_{it}^2, Hours_{it}\right]$$

Back - Rolling classification

Data - Wage inequality



$$\underbrace{\mathbb{V}_t\left[\log \widetilde{y}_{ijt}\right]}_{\text{A. Total variance}} = \underbrace{\sum_j \omega_{jt}\mathbb{V}_{jt}\left[\log \widetilde{y}_{ijt}\right]}_{\text{B. Within occupation}} + \underbrace{\sum_j \omega_{jt}\left(\mathbb{E}_{jt}\left[\log \widetilde{y}_{ijt}\right] - \mathbb{E}_t\left[\log \widetilde{y}_{ijt}\right]\right)^2}_{\text{C. Between occupation}}$$

- Red = High skill occupations, Blue = Low skill occupations
- 3 digit occupations - **Classified in 1980**

$$\mathbf{X}_{i,t} = \left[Year_t, NAICS1_{it}, Ed_{it}, Race_{it}, Sex_{it}, FirmSize_{it}, Exp_{it}, Exp^2_{it}, Hours_{it}\right]$$

Back - Rolling classification

Fact B. - Technology

- Input is a $J \times K$ normalized matrix of skill measures $\mathbf{A}$ from O*NET

1. Apply principal components with $K^* \ll K$

$$\mathbf{A}_{[J\times K]} = \widehat{\mathbf{A}}_{[J\times K^*]}\widehat{\mathbf{P}}_{[K^*\times K]} + \mathbf{U}_{[J\times K]}$$

2. To *name* skills, rotate principal components s.t. satisfy K^* orthogonality conditions

$$\mathbf{A}_{[J\times K]} = \left(\widehat{\mathbf{A}}_{[J\times K^*]}\Psi\right)\left(\Psi^{-1}\widehat{\mathbf{P}}_{[K^*\times K]}\right) + \mathbf{U}_{[J\times K]} \;\; \rightarrow \;\; \mathbf{A}^* = \widehat{\mathbf{A}}\Psi$$

$\implies$ Final skill 1, places a weight of 1 on $k = 1$, and zero on $k \in \{2, \ldots, K^*\}$

3. Use as K^* 'anchoring' skills those used by Acemoglu Autor (2011)

- Non-routine cognitive: Analytical - "Analyzing data / information"
- Non-routine cognitive: Interpersonal - "Maintaining relationships"
- Routine cognitive - "Importance of repeating the same tasks"
- Routine manual - "Controlling machines and processes"

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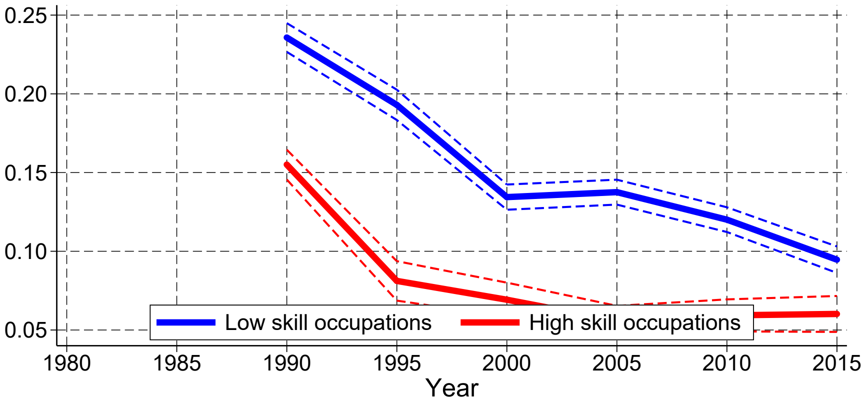
Decreasing size premium in low skill occ



1000+ employee firms associated with a 10 to 15 percent premium

$$\log Inc_{it} = \alpha + \beta^{\tau}_{Hours} \log Hours_{it} + \beta^{\tau}_{Exp} Exp_{it} + \beta^{\tau}_{Exp^2} Exp^2_{it} + \beta^{\tau}_{Size} Size_{it} \ldots$$

$$+\beta^{\tau}_{X} \left[Year_t, Race_{it}, NAICS1_{it}, Ed_{it}, Sex_{it}\right]$$

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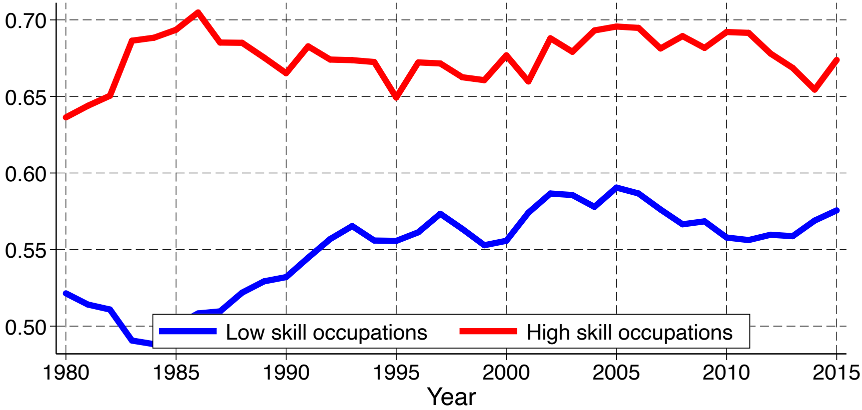
Increasing switching in low skill occ



Fraction of **male** workers experiencing $\left\{E_{March}, \ldots, U_m, \ldots, E_{March'}\right\}$

that swap **3-digit** occupations across $\left\{E_{March}, E_{March'}\right\}$

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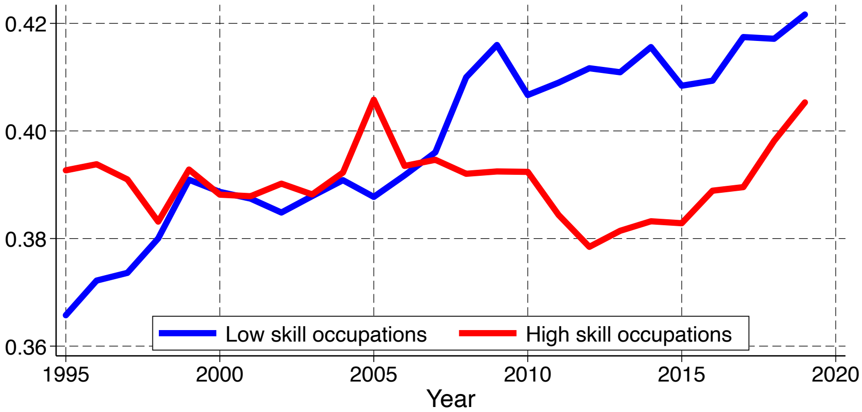
Increasing switching in low skill occ

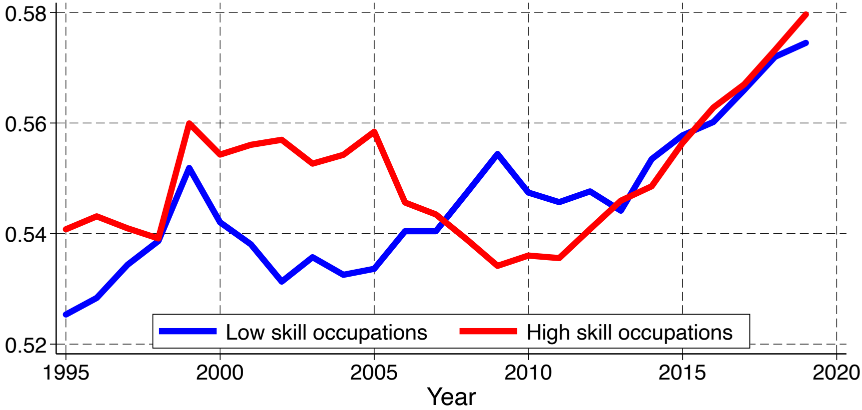


Fraction of **male** workers experiencing $\left\{E_{Month}, E_{Month+1}\right\}$

that swap **1-digit** occupations across $\left\{E_{Month}, E_{Month+1}\right\}$

▸ Back - Motivating empirics

Increasing switching in low skill occ



Fraction of **male** workers experiencing $\left\{E_{Month}, E_{Month+1}\right\}$

that swap **3-digit** occupations across $\left\{E_{Month}, E_{Month+1}\right\}$

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