Creating Confusion

Supplementary Online Appendix

Chris Edmond[*] Yang K. Lu[†]

February 2020

This appendix is organized as follows. In Appendix E we provide proofs of additional results omitted from the main text. In Appendix F we provide further details on the knife-edge case $c = 1$. In Appendix G we show that the expositional device of assuming that the coefficients in the reporters' strategy sum to one is without loss of generality.

[*]University of Melbourne. Email: ⟨cedmond@unimelb.edu.au⟩
[†]Hong Kong University of Science and Technology. Email: ⟨yanglu@ust.hk⟩

E Omitted proofs

In this appendix we provide proofs of results omitted from the main text. We first state and prove two supplementary lemmas used in the proof of Proposition 6 in the main text. We then provide proofs of Footnote 6 and Footnote 9 from the main text.

E.1 Supplementary lemmas

Supplementary Lemma 1. The total derivative of the reporters' equilibrium loss l^* with respect to α_x is strictly positive if and only if

$$F(k^*) := k^{*4} - 2k^{*3} + 2ck^* - c^2 > 0 \tag{E1}$$

Proof. Recall that $l^* = l(\delta^*; \alpha_x)$ where

$$l(\delta; \alpha_x) = \frac{(1-\lambda)}{(1-\delta)^2(1-\lambda)\alpha_x + \alpha_z} \tag{E2}$$

From this we obtain

$$\frac{dl^*}{d\alpha_x} > 0 \qquad \Leftrightarrow \qquad (1-\delta^*) - 2\alpha_x \frac{d\delta^*}{d\alpha_x} < 0 \tag{E3}$$

Equivalently, if and only if

$$\frac{d\delta^*}{d\alpha_x} > \frac{1}{2\alpha_x}(1-\delta^*) > 0 \tag{E4}$$

Now recall that in equilibrium the politician's manipulation depends on α_x only via the reporters' response coefficient, $\delta^*(\alpha_x) = \delta(k^*(\alpha_x))$, so that

$$\frac{d\delta^*}{d\alpha_x} = \delta'(k^*)\frac{dk^*}{d\alpha_x} \tag{E5}$$

So we can write condition (E4) as

$$\delta'(k^*)\frac{dk^*}{d\alpha_x} > \frac{1}{2\alpha_x}(1-\delta^*) > 0 \tag{E6}$$

Applying the implicit function theorem to the equilibrium condition (A2) from the main text we have

$$\frac{dk^*}{d\alpha_x} = \frac{\frac{\alpha_z}{(1-\lambda)\alpha_x}\frac{k^*}{\alpha_x}}{\frac{\alpha_z}{(1-\lambda)\alpha_x} - R'(k^*)} > 0 \tag{E7}$$

where $R(k)$ is defined in (A2) in the main text. Plugging this into (E6) and simplifying we get the equivalent condition

$$\frac{\alpha_z}{(1-\lambda)\alpha_x}\left(\delta'(k^*)k^* - \frac{1}{2}(1-\delta^*)\right) > -\frac{1}{2}(1-\delta^*)R'(k^*) \tag{E8}$$

Now observe from (A7) that

$$\delta'(k)k - \frac{1}{2}(1-\delta) = \frac{1}{2}\left(\frac{1}{c-k^2}\right)^2\left(k^3 - 3ck^2 + 3ck - c^2\right) \tag{E9}$$

and that using the formula for $R'(k)$ given in (A3) above we can calculate that

$$\frac{1}{2}(1-\delta)R'(k) = \frac{1}{2}\left(\frac{1}{c-k^2}\right)^2 R(k)\frac{1}{1-k}P(k) \tag{E10}$$

where $P(k)$ is also defined in (A3) above. Plugging these calculations back into (E8) gives

$$\frac{\alpha_z}{(1-\lambda)\alpha_x}\left(\frac{1}{2}\left(\frac{1}{c-k^{*2}}\right)^2\left(k^{*3} - 3ck^{*2} + 3ck^* - c^2\right)\right) > -\frac{1}{2}\left(\frac{1}{c-k^{*2}}\right)^2 R(k^*)\frac{1}{1-k^*}P(k^*) \tag{E11}$$

Canceling common terms gives the condition

$$\frac{\alpha_z}{(1-\lambda)\alpha_x}\left(k^{*3}-3ck^{*2}+3ck^*-c^2\right) > -R(k^*)\frac{1}{1-k^*}P(k^*) \tag{E12}$$

Using the equilibrium condition $L(k^*) = R(k^*)$ from (A2) and $\alpha = (1-\lambda)\alpha_x/\alpha_z$ gives

$$\frac{\alpha_z}{(1-\lambda)\alpha_x}\left(k^{*3}-3ck^{*2}+3ck^*-c^2\right) > -\frac{\alpha_z}{(1-\lambda)\alpha_x}\frac{k^*}{1-k^*}P(k^*) \tag{E13}$$

Using the definition of $P(k)$ and canceling more common terms gives the condition

$$k^{*4}-2k^{*3}+2ck^*-c^2 > 0 \tag{E14}$$

□

Supplementary Lemma 2. Define

$$F(k^*) := k^{*4}-2k^{*3}+2ck^*-c^2 \tag{E15}$$

(i) If $c > 1$, then $F(k^*) < 0$;

(ii) If $c < 1$, there is an interval $(\underline{k}, \overline{k})$ with $0 < \underline{k} < \overline{k} < 1$ such that $F(k) > 0$ for $k \in (\underline{k}, \overline{k})$ and $F(k) \leq 0$ otherwise. Moreover, the cutoffs are on either side of c so that $0 < \underline{k} < c < \overline{k} < 1$.

Proof. Write $F(k) = J(k;c) - G(k)$ where $J(k;c) := 2ck - c^2$ and $G(k) := 2k^3 - k^4$. Observe that $G(0) = 0$, $G(1) = 1$, $G(k) < k$ for all k; $G'(k) = 2k^2(3-2k) \geq 0$ with $G'(0) = 0$ and $G'(1) = 2$; and $G''(k) = 12k(1-k) \geq 0$ so that $G'(k) \leq G'(1) = 2$ for all k. Further observe that $J(0;c) = -c^2 < 0$, $J(1;c) = 2c - c^2 \leq 1$ (with equality if $c = 1$) and $J'(k;c) = 2c > 0$ for all k so that $J(k;c) \leq J(1;c) = 2c - c^2 \leq 1$ for all k, c. These imply $F(0) = J(0;c) - G(0) = -c^2 < 0$ and $F(1) = J(1;c) - G(1) = 2c - c^2 - 1 \leq 0$ (with equality if $c = 1$); $F'(k) = J'(k;c) - G'(k) = 2c - G'(k)$ and $F''(k) = -G''(k) \leq 0$. Since $G'(k) \leq 2$ we have

$$F'(k) = J'(k;c) - G'(k) = 2c - G'(k) \geq 2c - 2 = 2(c-1) \tag{E16}$$

For part (i) $c > 1$. Then $F'(k) \geq 2(c-1) > 0$ so $F(k)$ is strictly increasing from $F(0) = -c^2 < 0$ to $F(1) = 2c - c^2 - 1 < 0$ so that $F(k) < 0$ for all k.

For part (ii) $c < 1$. Then since $G'(k)$ is monotone increasing from $G'(0) = 0$ to $G'(1) = 2$ there is a unique critical point $\tilde{k}$ such that

$$F'(\tilde{k}) = 0 \qquad \Leftrightarrow \qquad 2c = G'(\tilde{k}) \tag{E17}$$

Since $F''(k) \leq 0$, this critical point maximizes $F(k)$ hence

$$F(k) \leq \max_{k\in[0,1]} F(k) = F(\tilde{k}) \tag{E18}$$

and observe that if we take $k = c < 1$ (which is feasible since here $c < 1$) then we have

$$F(c) = J(c;c) - G(c) = 2c^2 - c^2 - G(c) = c^2 - 2c^3 + c^4 = c^2(1 - 2c + c^2) > 0 \tag{E19}$$

so that indeed

$$F(\tilde{k}) \geq F(c) > 0 \tag{E20}$$

Hence for $c < 1$ there exist k such that $F(k) > 0$. More precisely, the function $F(k)$ increases from $F(0) = -c^2 < 0$ to a lower cutoff $\underline{k} \in (0, \tilde{k})$ defined by $F(\underline{k}) = 0$. The function $F(k)$ keeps increasing until it reaches the critical point $\tilde{k}$ at which $F'(\tilde{k}) = 0$ and $F(\tilde{k}) > 0$. From there $F(k)$ decreases, crossing zero again at a higher cutoff $\overline{k} \in (\tilde{k}, 1)$ defined by $F(\overline{k}) = 0$ and keeps decreasing until $F(1) = 2c - c^2 - 1 < 0$ (since $c < 1$).

So for $c < 1$ there is an interval $(\underline{k}, \overline{k})$ with $0 < \underline{k} < \overline{k} < 1$ such that $F(k) > 0$ for $k \in (\underline{k}, \overline{k})$ and $F(k) \leq 0$ otherwise. For $c < 1$ these critical points are defined by the roots of $F(k;c) = 0$. Observe that since $F(c) > 0$ yet $\underline{k}$ is the first k for which $F(k) = 0$ it must be the case that $\underline{k} < c$. Likewise since $F(\overline{k}) = 0$ it must also be the case that $\overline{k} > c$. In short, the cutoffs are on either side of c so that $0 < \underline{k} < c < \overline{k} < 1$. □

E.2 Additional footnotes

Proof of Footnote 6.

Suppose that $\lambda < 0$ and $c < c^*_{nm}(\alpha)$ so that $k^* < k^*_{nm}$. We can rewrite the condition (B19) for the politician's manipulation to backfire as

$$\frac{k^{*2}}{c - k^*} \frac{(1-k^*)^2}{k^*_{nm} - k} < 1 - \lambda(k^*_{nm} + k^*). \tag{E21}$$

Using the equilibrium condition (A2) and the politician's best response (17), we have:

$$\frac{k^{*2}}{c - k^*} = \alpha \frac{c(1-k^*)k^*}{(c - k^{*2})^2} = \alpha\delta^* \left(1 + \frac{k^*}{1-k^*}\delta^*\right). \tag{E22}$$

As $c \to 0$, the RHS of (E21) converges to

$$\lim_{c \to 0} \left(1 - \lambda(k^*_{nm} + k^*)\right) = 1 - \lambda \frac{\alpha}{\alpha + 1} \tag{E23}$$

since $k^* \to 0$ as $c \to 0$. Recall that as $c \to 0$, $\delta^* \to 1$, the LHS of (E21) converges to

$$\lim_{c \to 0} \left(\frac{k^{*2}}{c - k^*} \frac{(1-k^*)^2}{k^*_{nm} - k}\right) = \alpha \frac{1}{\frac{\alpha}{\alpha+1}} = \alpha + 1. \tag{E24}$$

Therefore, the condition (E21) must hold when c is small enough if

$$\alpha + 1 < 1 - \lambda \frac{\alpha}{\alpha + 1}. \tag{E25}$$

Since $\alpha = (1-\lambda)\alpha_x/\alpha_z$, the inequality above can be rewritten as

$$\alpha_x + \alpha_z < (\alpha_x - \alpha_z)\lambda. \tag{E26}$$

Given that $\lambda < 0$, a necessary condition for the inequality above to hold is $\alpha_x < \alpha_z$. When this is the case, the inequality above is equivalent to

$$\lambda < -\frac{\alpha_x + \alpha_z}{\alpha_z - \alpha_x} < -1. \tag{E27}$$

In sum, when $\alpha_x < \alpha_z$, for each λ satisfying the inequality (E27), there must exist a cutoff $\underline{c}^*$ such that for all $c < \underline{c}^*$, the condition (E21) for the politician's manipulation to backfire holds. Finally, the cutoff $\underline{c}^*$ must be lower than $c^*_{nm}(\alpha)$ so that $c < \underline{c}^*$ is sufficient for $k^* < k^*_{nm}$. □

Proof of Footnote 9.

The total derivative of v^* with respect to α_x can be written as

$$\frac{dv^*}{d\alpha_x} = v'(k^*)\frac{\partial k^*}{\partial \alpha_x} + \frac{\partial v(k^*; \alpha_x)}{\partial \alpha_x}. \tag{E28}$$

Since

$$v'(k^*) = -2\frac{\lambda}{1-\lambda}\left(\frac{k^*}{\alpha_x}\right) \tag{E29}$$

according to Lemma 5 and

$$\frac{\partial v(k^*; \alpha_x)}{\partial \alpha_x} = -\left(\frac{k^*}{\alpha_x}\right)^2 < 0, \tag{E30}$$

we can then write the total derivative (E28) as

$$\frac{dv^*}{d\alpha_x} = -2\frac{\lambda}{1-\lambda}\left(\frac{k^*}{\alpha_x}\right)\frac{\partial k^*}{\partial \alpha_x} - \left(\frac{k^*}{\alpha_x}\right)^2 \tag{E31}$$

which is negative if

$$-2\frac{\lambda}{1-\lambda} < \frac{J_1}{J_2} \tag{E32}$$

where

$$J_1 := (c - k^{*2})^2 - 4k^{*2}(c - k^*)(1 - k^*)$$
$$J_2 := (c - k^*)(1 - k^*)(c - k^{*2})$$

Observe that as $\alpha_x \to 0$ such that $k^* \to 0$ the ratio $J_1/J_2 \to 1$. The derivative of J_1/J_2 with respect to k^* has the same sign as

$$\frac{\partial J_1}{\partial k^*}J_2 - \frac{\partial J_2}{\partial k^*}J_1 \geq 2\sqrt{c}(1 - k^*)(c - k^*)\left((k^{*2} - 2\sqrt{c}k^* + c)^2 + 4\sqrt{c}k^{*2}(\sqrt{c} - 1)^2\right) \geq 0 \tag{E33}$$

So J_1/J_2 is increasing in k^*. From Lemma 3, we know that k^* is increasing in α and in turn α_x. So J_1/J_2 is increasing in α_x and hence is bounded below by 1.

If $\lambda > -1$, the LHS of (E32) is strictly lower than 1. Therefore, the condition (E32) for v^* to be strictly decreasing in α_x must hold.

If $\lambda < -1$, observe that as $\alpha_x \to \infty$ such that $k^* \to \min(c, 1)$, $J_2 \to 0$ and $J_1 \to (c - k^{*2})^2 > 0$. So the RHS of (E32) approaches to positive infinity. Since the RHS of (E32) is also increasing in α_x, there must exist a cutoff in α_x such that the condition (E32) holds for α_x higher than the cutoff. □

F Knife-edge case $c = 1$

Preliminaries. There is no issue with $c = 1$ if the composite parameter $\alpha \leq 4$. The issues with $c = 1$ arise only if $\alpha > 4$. To see this, first recall from Lemma 1 that if $\alpha > 1$ the reporters' best response $k(\delta; \alpha)$ is increasing in δ on the interval $[0, \hat{\delta}(\alpha)]$ and obtains its maximum at $\delta = \hat{\delta}(\alpha) = 1 - 1/\sqrt{\alpha} \in (0, 1)$. At the maximum, the reporters' best response takes on the value $k(\hat{\delta}(\alpha); \alpha) = \sqrt{\alpha}/2$. Hence for $\alpha > 4$ the maximum value exceeds 1. Moreover, by continuity of the best response in δ if $\alpha > 4$ there is an interval of δ such that $k(\delta; \alpha) > 1$. The boundaries of this interval $(\underline{\delta}(\alpha), \overline{\delta}(\alpha))$ are given by the roots of $k(\delta; \alpha) = 1$, which work out to be

$$\underline{\delta}(\alpha)\,,\, \overline{\delta}(\alpha) = \frac{1}{2}\Big(1 \pm \sqrt{1 - (4/\alpha)}\Big), \qquad \alpha \geq 4 \tag{F1}$$

Observe that this interval is symmetric and centred on $\frac{1}{2}$ with a width of

$$\overline{\delta}(\alpha) - \underline{\delta}(\alpha) = \sqrt{1 - (4/\alpha)} \geq 0, \qquad \alpha \geq 4 \tag{F2}$$

If $\alpha = 4$, we have $\underline{\delta}(4) = \overline{\delta}(4) = \frac{1}{2}$ but as α increases the width of the interval $(\underline{\delta}(\alpha), \overline{\delta}(\alpha))$ expands around $\frac{1}{2}$ with the boundaries $\underline{\delta}(\alpha) \to 0^+$ and $\overline{\delta}(\alpha) \to 1^-$ as $\alpha \to \infty$. Now recall from Proposition 1 that only $k \in [0, \min(c, 1)]$ and $\delta \in [0, 1]$ are candidates for an equilibrium. So if $\alpha > 4$ then none of the values of $\delta \in (\underline{\delta}(\alpha), \overline{\delta}(\alpha))$ are candidates for an equilibrium.

Cost of manipulation, $c \neq 1$. Now consider the politician's best response $\delta(k; c)$ parameterized by $c \neq 1$ and suppose $\alpha > 4$. When $c \neq 1$, the politician's objective always depends on δ over the entire support $k \in [0, \min(c, 1)]$. As proved in Proposition 1, there is a unique intersection between the politician's and the reporters' best responses. As illustrated below, if $c < 1$ the politician's best response $\delta(k; c)$ must lie above $\delta(k; 1) = k/(1 + k)$ and hence the equilibrium point k^*, δ^* must be on the "upper branch" of $k(\delta; \alpha)$ where $\delta^* > \overline{\delta}(\alpha)$. But for the same value of α and instead $c > 1$ the equilibrium point k^*, δ^* must be on the "lower branch" of $k(\delta; \alpha)$ where $\delta^* < \underline{\delta}(\alpha)$ because the politician's best response $\delta(k; c > 1)$ lies below $\delta(k; 1) = k/(1 + k)$.

Knife-edge case, $c = 1$. Now consider the case $c = 1$ exactly. The relevant part of the politician's objective becomes

$$B(\delta, k) - C(\delta) = (k^2 - 1)\delta^2 + 2k(1 - k)\delta + (1 - k)^2 \tag{F3}$$

When $k \neq 1$, the politician's best response is $\delta(k; 1) = (k - k^2)/(1 - k^2) = k/(1 + k)$, which is increasing in k and approaches $1/2$ as $k \to 1$. But when $k = 1$, the politician's objective is independent of δ and in turn the politician is indifferent in the choice of δ. The equilibrium $(k^* = 1, \delta^*)$ is thus entirely determined by the reporters' best response. If $\alpha < 4$, the reporters' best response $k(\delta; \alpha) < 1$ so that $k^* = 1$ is never an equilibrium. If $\alpha = 4$, there is a unique equilibrium determined by the maximum of the reporters' best response $(k^* = 1, \delta^* = 1/2)$. If $\alpha > 4$, there are two equilibria corresponding to the two roots of $k(\delta; \alpha) = 1$: namely $(k^* = 1, \delta^* = \underline{\delta}(\alpha))$ and $(k^* = 1, \delta^* = \overline{\delta}(\alpha))$.

Further intuition for large changes in manipulation near $c = 1$. Now consider the sensitivity of the equilibrium amount of manipulation to changes in c near $c = 1$. Recall that, taking the reporters' k as given, the politician chooses manipulation δ to maximize

$$V(\delta, k) = \frac{1}{\alpha_z}\left(B(\delta, k) - C(\delta)\right) + \frac{1}{\alpha_x}k^2 \tag{F4}$$

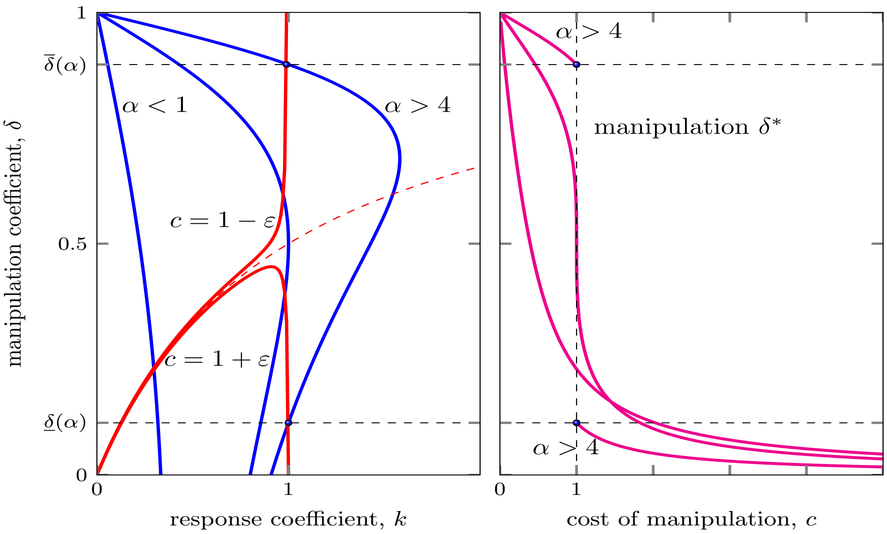
with benefit $B(\delta, k) = (k\delta + 1 - k)^2$ and cost of manipulation $C(\delta) = c\delta^2$.

Now consider an environment where the reporters are inclined to be very responsive to their signals, $\alpha \to \infty$ so that $k \to \min(c, 1)$. First, suppose that $c > 1$ so that $k \to 1$. Then the relevant part of the politician's objective simplifies to

$$B(\delta, 1) - C(\delta) = (1 - c)\delta^2 \tag{F5}$$

so that for any $c > 1$ the politician will choose $\delta = 0$. Next, suppose instead that $c < 1$ so that $k \to c$. In this case the relevant part of the politician's objective simplifies to

$$B(\delta, c) - C(\delta) = -c(1 - c)\delta^2 + 2c(1 - c)\delta + (1 - c)^2 \tag{F6}$$



Discontinuity at $c = 1$ and jump in the amount of manipulation δ^*

The left panel shows the reporters' best response $k(\delta;\alpha)$ for $\alpha < 1$, $\alpha = 4$ and $\alpha > 4$ (blue) and the politician's best response $\delta(k;c)$ for $c = 1 - \varepsilon$, $c = 1$, and $c = 1 + \varepsilon$ (red). For $\alpha > 4$, in the limit as $c \to 1^-$ the equilibrium is at $k^* = 1, \delta^* = \overline{\delta}(\alpha)$ but in the limit as $c \to 1^+$ the equilibrium is at $k^* = 1, \delta^* = \underline{\delta}(\alpha)$. For $\alpha > 4$ and $c = 1$ exactly both of these are equilibria because for this knife-edge special case the politician is indifferent between $\underline{\delta}(\alpha)$ and $\overline{\delta}(\alpha)$. The right panel shows the equilibrium manipulation δ^* as a function of c for $\alpha < 1$, $\alpha = 4$ and $\alpha > 4$. For $\alpha \leq 4$, the manipulation δ^* is continuous in c. But for $\alpha > 4$ the manipulation jumps discontinuously at $c = 1$. In the limit as $\alpha \to \infty$ the boundaries $\underline{\delta}(\alpha) \to 0^+$ and $\overline{\delta}(\alpha) \to 1^+$ so that the manipulation jumps by the maximum possible amount, from $\delta^* = 0$ if $c < 1$ to $\delta^* = 1$ if $c > 1$.

so that for any $c < 1$ the politician will choose $\delta = 1$. In short, as $\alpha \to \infty$, the politician's manipulation is a *step function* in c, with $\delta = 1$ for all $c < 1$ and $\delta = 0$ for all $c > 1$.

What is the meaning of $c = 1$? So given that the amount of manipulation can be extremely sensitive to c near $c = 1$, what does $c = 1$ mean? Recall that in the politician's objective (5) the gross benefit $\int_0^1 (a_i - \theta)^2 \, di$ has a coefficient normalized to 1. If instead we had written the objective with $b \int_0^1 (a_i - \theta)^2 \, di$ for some $b > 0$ then throughout the analysis the relevant parameter would be the cost/benefit ratio c/b and the critical point would be where the cost/benefit ratio is $c/b = 1$. In this parameterization, the politician's equilibrium manipulation is extremely sensitive to changes in either c or b in the vicinity of $c/b = 1$. With α high and costs and benefits *evenly poised*, a small decrease in b or small increase in c would lead to a large reduction in manipulation.

G Coefficients sum to one

In this appendix we show that writing the reporters' linear strategy as $a(x_i) = kx_i + (1-k)z$ is without loss of generality. Suppose that the reporters' linear strategy is

$$a_i = \beta_0 + \beta_1 x_i + \beta_2 z$$

for some coefficients $\beta_0, \beta_1, \beta_2$. We will show that in any linear equilibrium $\beta_0 = 0$ and $\beta_1 + \beta_2 = 1$. With this strategy, the aggregate A is

$$A = \beta_0 + \beta_1 y + \beta_2 z$$

The politician's problem is then to choose y to maximize

$$\int_0^1 (a_i - \theta)^2 \, di - c(y-\theta)^2 = (\beta_0 + \beta_1 x_i + \beta_2 z - \theta)^2 + \frac{1}{\alpha_x}\beta_1^2 - c(y-\theta)^2$$

The solution to this problem is

$$y = \gamma_0 + \gamma_1 \theta + \gamma_2 z$$

where

$$\gamma_0 = \frac{\beta_0 \beta_1}{c - \beta_1^2} \tag{G1}$$

$$\gamma_1 = \frac{c - \beta_1}{c - \beta_1^2} \tag{G2}$$

$$\gamma_2 = \frac{\beta_1 \beta_2}{c - \beta_1^2} \tag{G3}$$

But if the politician has the strategy $y = \gamma_0 + \gamma_1\theta + \gamma_2 z$, the reporters' posterior expectation of θ is

$$\begin{aligned}
\mathbb{E}[\theta \,|\, x_i] &= \frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z}\left(\frac{1}{\gamma_1}(x_i - \gamma_2 z) - \frac{\gamma_0}{\gamma_1}\right) + \frac{\alpha_z}{\gamma_1^2\alpha_x + \alpha_z} z \\
&= \frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z} x_i + \frac{\alpha_z - \gamma_1 \alpha_x \gamma_2}{\gamma_1^2 \alpha_x + \alpha_z} z - \frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z}\gamma_0
\end{aligned}$$

And the equilibrium strategy of an individual reporter then satisfies

$$\begin{aligned}
a_i &= \lambda \mathbb{E}[A \,|\, x_i] + (1-\lambda)\mathbb{E}[\theta \,|\, x_i] \\
&= \lambda \beta_1 \mathbb{E}[y \,|\, x_i] + (1-\lambda)\mathbb{E}[\theta \,|\, x_i] + \lambda \beta_2 z + \lambda \beta_0 \\
&= (\lambda \beta_1 \gamma_1 + (1-\lambda))\mathbb{E}[\theta \,|\, x_i] + \lambda(\beta_1 \gamma_2 + \beta_2) z + \lambda(\beta_1 \gamma_0 + \beta_0)
\end{aligned}$$

Matching coefficients with $a_i = \beta_0 + \beta_1 x_i + \beta_2 z$ we then have

$$\beta_0 = -(\lambda\beta_1\gamma_1 + (1-\lambda))\frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z}\gamma_0 + \lambda(\beta_1 \gamma_0 + \beta_0) \tag{G4}$$

$$\beta_1 = \quad (\lambda\beta_1\gamma_1 + (1-\lambda))\frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z} \tag{G5}$$

$$\beta_2 = \quad (\lambda\beta_1\gamma_1 + (1-\lambda))\frac{\alpha_z - \gamma_1 \alpha_x \gamma_2}{\gamma_1^2 \alpha_x + \alpha_z} + \lambda(\beta_1\gamma_1 + \beta_2) \tag{G6}$$

Now observe that equations (G1) and (G4) together imply that the intercepts are $\beta_0 = \gamma_0 = 0$. Now observe from (G2)-(G3) and (G5)-(G6) that $\gamma_1 + \gamma_2 = 1$ implies $\beta_1 + \beta_2 = 1$ and vice-versa. So in one equilibrium the reporters' strategy takes the form $a_i = kx_i + (1-k)z$ where $k = \beta_1$ and the politician's strategy takes the form $y = (1-\delta)\theta + \delta z$ where $\delta = \gamma_2$. Hence from (G3) and (G5) we can write

$$\delta = \frac{k - k^2}{c - k^2}, \qquad k = \frac{(1-\delta)\alpha}{(1-\delta)^2\alpha + 1}$$

where $\alpha := (1-\lambda)\alpha_x/\alpha_z$. These are the same as the best response formulas equations (17) and (24) in the main text and from Proposition 1 we know that there is a unique pair k^*, δ^* satisfying these conditions.