# 316-632 INTERNATIONAL MONETARY ECONOMICS **Final samples**

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The final will last 120 minutes and has two questions. Both questions are worth 60 marks. Within each question there are a number of parts and the weight given to each part will also be indicated.

Here are two sample questions.

**Question 1.** *International relative prices in the Lucas model* (60 marks). Let there be countable dates, $t = 0, 1, 2...$ and let a state of nature be indexed by $z_t$. A history is a vector $z^t = (z_0, z_1, ..., z_t)$. The unconditional probability of a history $z^t$ being realized as of date zero is denoted $\pi_t(z^t)$. The initial state $z_0$ is known as of date zero. Let there be 2 countries with $i = 1, 2$. There are two goods that are both traded internationally. Good $a$ only comes from country 1 and comes in amount $y_t^1(z^t)$ while good $b$ only comes from country 2 and comes in amount $y_t^2(z^t)$. Each good is consumed by both countries. Preferences in the two countries are identical and given by the expected utility function

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t U\{G[a_t^i(z^t), b_t^i(z^t)]\} \pi_t(z^t), \qquad 0 < \beta < 1$$

Period utility is given by a CRRA function over a CES aggregate of the $a$ and $b$ goods, namely

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \qquad \sigma > 0$$

and

$$c = G(a, b) \equiv \left[\mu a^{\frac{\gamma-1}{\gamma}} + (1-\mu) b^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \qquad 0 < \mu < 1 \text{ and } \gamma > 0$$

In this CES aggregate, the two goods become perfect substitutes as $\gamma \to \infty$ and become perfect complements as $\gamma \to 0$. The world-wide resource constraints are

$$\begin{aligned} a_t^1(z^t) + a_t^2(z^t) &= y_t^1(z^t) \\ b_t^1(z^t) + b_t^2(z^t) &= y_t^2(z^t) \end{aligned}$$

(a) (10 marks): Set up the equivalent social planner's problem with welfare weights $\omega_i > 0$. Let

$$\begin{aligned} Q_t^1(z^t) &\equiv \beta^t \pi_t(z^t) q_t^1(z^t) \\ Q_t^2(z^t) &\equiv \beta^t \pi_t(z^t) q_t^2(z^t) \end{aligned}$$

denote the planner's Lagrange multipliers for the two sets of resource constraints. Explain how the planner's Lagrange multipliers relate to market prices in the corresponding decentralized model. Derive first order conditions that characterize the solution to the planner's problem.

(b) (30 marks): Solve the model for the allocations $\{a_t^1(z^t), a_t^2(z^t), b_t^1(z^t), b_t^2(z^t)\}$ and the Lagrange multipliers $\{q_t^1(z^t), q_t^2(z^t)\}$. Your answers should be expressed in terms of the planner's welfare weights and the exogenous stochastic processes for the endowments, $\{y_t^1(z^t), y_t^2(z^t)\}$. [*Hint*: it will simplify the algebra is you use the fact that $G(a, b)$ exhibits constant returns so that $G(a, b) = aG(1, \frac{b}{a})$ for $a > 0$].

(c) (20 marks): The terms of trade facing country $i = 1$ may be defined by

$$p_t(z^t) \equiv \frac{q_t^2(z^t)}{q_t^1(z^t)}$$

Using your answers from part (b), solve for the terms of trade. Explain how the dynamics of the terms of trade depend on the exogenous stochastic processes for $\{y_t^1(z^t), y_t^2(z^t)\}$. How does the volatility of the log terms of trade depend on the degree of substitution $\gamma$ between the two goods? Finally, explain how the real exchange rate is determined in this model. Does PPP hold in this model? Why or why not? How does your answer regarding the real exchange rate change (if at all) when preferences are asymmetric. To be concrete, suppose that preferences in country $i = 1$ are

$$G_1(a, b) \equiv \left[\mu a^{\frac{\gamma-1}{\gamma}} + (1-\mu) b^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \qquad \frac{1}{2} < \mu < 1$$

but preferences in country $i = 2$ are instead

$$G_2(a, b) \equiv \left[(1-\mu) a^{\frac{\gamma-1}{\gamma}} + \mu b^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \qquad \frac{1}{2} < \mu < 1$$

Explain, intuitively, how this impacts on the determinants of the real exchange rate.

**Question 2.** *Exchange rate risk premia* (60 marks). Suppose that we write the covered and uncovered interest parity relationships as

(CIP) $$i_t - i_t^* = f_t - e_t$$

(UIP) $$i_t - i_t^* = \mathsf{E}_t\left\{\Delta e_{t+1}\right\}$$

where $f_t$ denotes the log forward exchange rate, $e_t$ denotes the log spot exchange rate (and a rise in $e_t$ is a depreciation of the home currency), $i_t$ denotes the nominal interest rate on home bonds and $i_t^*$ denotes the nominal interest rate on foreign bonds.

(a) (5 marks): Give an intuitive explanation for why we might expect these relationships to hold.

(b) (5 marks): If we know that covered interest parity does in fact hold, explain how a time-series regression of the form

$$e_{t+1} - e_t = \alpha + \beta(f_t - e_t) + \text{error}$$

can be used to test the uncovered interest parity relationship. If uncovered interest parity holds, what coefficient would we expect to obtain for $\beta$? If you are told that typical estimates are $\beta < 0$, explain why this is puzzling from the point of view of simple economic theory.

(c) (15 marks): Define an exchange rate risk premium $\rho_t$ by

$$\rho_t \equiv f_t - \mathsf{E}_t\{e_{t+1}\}$$

An implication of $\beta < 0$ is that the exchange rate risk premium must not be constant. Explain why. [*Hint*: here and throughout this question, you may assume that there is no difference between the true and the estimated $\beta$].

(d) (15 marks): If $\beta < 0$, does $\mathsf{Cov}\{\mathsf{E}_t\{\Delta e_{t+1}\}, \rho_t\}$ have to be positive or negative? Explain your answer either way. Does this make intuitive economic sense? Why or why not?

(e) (20 marks): If $\beta < 0$, explain why

$$\mathsf{Var}\{\rho_t\} > \mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\}$$

Empirically, is this a stringent requirement? Why or why not? Now decompose interest differentials as

$$i_t - i_t^* = f_t - e_t = f_t - \mathsf{E}_t\{e_{t+1}\} + \mathsf{E}_t\{e_{t+1}\} - e_t$$

Given your previous answers, if $\beta < 0$, how much of the volatility in interest differentials is accounted for by volatility in expected exchange rate depreciations? How much is accounted for by volatility in the risk premium?

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