316-632 International Monetary Economics

Problem Set #2 *Due 6 September, 2004*

This problem set is designed to introduce you to solving simple dynamic models on a computer.

Consider the social planning problem of maximizing utility

$$\mathsf{E}_0\left\{\sum_{t=0}^{\infty}\beta^t[U(c_t)+V(\ell_t)]\right\},\qquad 0<\beta<1$$

subject to a resource constraint

$$c_t+i_t=z_tF(k_t,n_t)$$

a constraint on the time endowment

$$n_t+\ell_t=1$$

and the following law of motion for capital accumulation

$$k_{t+1}=(1-\delta)k_t+i_t-\phi\left(\frac{i_t}{k_t}\right)k_t,\qquad 0<\delta<1$$

where ϕ is a strictly increasing strictly convex cost of adjustment function with the properties $\phi(\delta)=\phi'(\delta)=0$ and where δ is the depreciation rate of physical capital. Finally, let log technology follow an AR(1),

$$\log(z_{t+1})=\rho\log(z_t)+\varepsilon_{t+1},\qquad 0<\rho<1$$

where ε_{t+1} is Gaussian white noise with initial realization z_0 given.

Other than the costs of capital adjustment, this is just the same model as in the notes.

Question 1. Derive first order conditions for the planner's problem and show how these characterize optimal choices of consumption, leisure, employment, and investment.

Question 2. Let the unconditional mean of the stochastic technology shock be $\bar{z}=1$. Show how to characterize the non-stochastic steady state of the model. Now let the functional forms be

$$\begin{aligned}U(c) &= \log(c)\\ V(\ell) &= \log(\ell)\\ F(k,n) &= k^{\theta}n^{1-\theta}\\ \phi\left(\frac{i}{k}\right) &= \frac{1}{2}\left(\frac{i}{k}-\delta\right)^2\end{aligned}$$

and let the parameters of the model be given by

Symbol	Meaning	Value
β	time discount factor	0.99
θ	capital's share in national output	0.33
δ	depreciation rate of physical capital	0.04
ρ	serial correlation of technology shock	0.95

Use a programming language like Matlab to solve for the non-stochastic steady state with these functional forms and parameter values. Matlab is available through the Department's Citrix server

⟨`http://hearn.ecom.unimelb.edu.au/Citrix/MetaFrameXP/default/login.asp`⟩

Question 3. Log-linearize the model around the non-stochastic steady state. That is, if the model is written in the form

$$\begin{aligned} 0 &= AX_t + BX_{t-1} + CY_t + DZ_t \\ 0 &= \mathsf{E}_t\{FX_{t+1} + GX_t + HX_{t-1} + JY_{t+1} + KY_t + LZ_{t+1} + MZ_t\} \\ Z_{t+1} &= NZ_t + \varepsilon_{t+1} \end{aligned}$$

provide explicit solutions for each of the coefficients, $A, B, C, ..., N$. In these equations, X_t contains the endogenous state variables, Y_t contains the control variables, and Z_t contains the exogenous state variables. As part of your answer, you will need to explain exactly which variables from the model are in each of X_t, Y_t and Z_t.

Question 4. Guess that a solution takes the form

$$\begin{aligned} X_t &= PX_{t-1} + QZ_t \\ Y_t &= RX_{t-1} + SZ_t \end{aligned}$$

for unknown coefficient matrices P, Q, R, S. Show that solving this model reduces to solving a quadratic equation in P. Solve for the value of P using the answers from Questions 2 and 3, then recover the Q, R, S values. Again, this is easy to do in Matlab.

Question 5. Use your answers to simulate the effect of a one-time shock to the level of productivity. That is, set the value of $\varepsilon_0 = 1$ and $\varepsilon_t = 0$ for $t \geq 1$ and trace out the effects on productivity, consumption, employment, investment etc. Graph your answers for $t = 0, 1, ..., 50$.

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