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International consumption correlations

- In an Arrow-Debreu model with complete markets, cross-country consumption growth correlations should be "high". In such a model, idiosyncratic country-specific income shocks should be perfectly pooled so that each country is exposed only to aggregate (or "world") income shocks. To see why complete markets models have this implication, imagine that each country has a representative consumer. If each consumer has an intertemporal marginal rate of substitution equal to the same price ratio (which in turn depends only on aggregate income), cross-country marginal rates of substitution should be **perfectly correlated**. Moreover, if we make the typical separability assumptions about the period utility function, the perfect correlation of marginal rates of substitution should directly imply the perfect correlation of consumption growth.
- Specifically, suppose that we have complete markers with histories s^t and $s \in \mathcal{S}$ states of nature each period. Then the key first order condition for country i will be something like

  $$U'[c_t^i(s^t)] = \lambda^i p_t(s^t), \qquad \text{each } s^t$$

  where, depending on the interpretation, $p_t(s)$ is either an intertemporal price or the relevant Lagrange multiplier of the planner while λ^i is either the single Lagrange multiplier on an intertemporal budget constraint or the reciprocal of the planner's welfare weight. For any two countries i and j we have

  $$\frac{U'[c_t^i(s^t)]}{\lambda^i} = p_t(s^t) = \frac{U'[c_t^j(s^t)]}{\lambda^j}, \qquad \text{each } s^t$$

  Up to a multiplicative constant, the marginal utilities are equal across pairs of countries. Moreover, comparing any two dates gets rid of the constants so that marginal rates of substitution are equalized

  $$\frac{U'[c_{t+1}^i(s^t,s')]}{U'[c_t^i(s^t)]} = \frac{p_{t+1}(s^t,s')}{p_t(s^t)} = \frac{U'[c_{t+1}^j(s^t,s')]}{U'[c_t^j(s^t)]}, \qquad \text{each } s^t \text{ and } s'$$

  Hence marginal rates of substitution will be perfectly correlated. Moreover, if we have the usual CRRA utility function, this implies consumption growth should be equal state by state

$$\frac{c^i_{t+1}(s^t, s')}{c^i_t(s^t)} = \frac{c^j_{t+1}(s^t, s')}{c^j_t(s^t)}, \qquad \text{each } s^t \text{ and } s'$$

If so, domestic consumption growth should not depend on shocks that are specific to the domestic country.

- Obstfeld and Rogoff claim that this is a corollary of the Feldstein-Horioka and home bias puzzles. Their reason for saying this is that because of the FH and home bias puzzles, we already know that the world is not as perfectly-integrated as complete markets theory would assume. So maybe it should be no great extra surprise to learn that consumption patterns are not so well integrated either. Given this, the main reason for independently considering consumption correlations is that in standard models, the behavior of consumption has **direct** implications for consumer welfare.
- Obstfeld and Rogoff document that in Penn World Tables (PWT) data for 1973-1992, the largest bilateral consumption correlation coefficient is 0.68 (this is for France/Japan). The average bilateral correlation coefficient in the same data is only 0.40. In each case, this is a long way short of the correlation of 1.00 that simple theory predicts.
- More sophisticated theory that allows for non-separabilities in utility does not predict that consumption correlations should be 1.00, however. For example, if utility depends on consumption $c^i_t(s^t)$ and leisure $\ell^i_t(s^t)$, then theory predicts only that the marginal rates of substitution are perfectly correlated. Say,

$$\frac{U_c[c^i_{t+1}(s^t, s'), \ell^i_{t+1}(s^t, s')]}{U_c[c^i_t(s^t), \ell^i_t(s^t)]} = \frac{p_{t+1}(s^t, s')}{p_t(s^t)} = \frac{U_c[c^j_{t+1}(s^t, s'), \ell^j_{t+1}(s^t, s')]}{U_c[c^j_t(s^t), \ell^j_t(s^t)]}$$

It turns out that this still implies a large bilateral consumption correlation. For example, in their calibrated complete markets model, Backus-Kehoe-Kydland find that the bilateral consumption correlation should not be 1.00 but should be about 0.90. So empirical consumption correlations around 0.40-0.70 still seem too low.
- Obstfeld and Rogoff claim that there trade costs should do as well here, but provide no calculations to back this claim up.
- The problem with Obstfeld and Rogoff's argument is the following (this follows Engel's discussion): trade costs give rise to national price levels that differ, say price levels $p^i_t(s^t)$ and

$p_t^j(s^t)$. But following Backus-Smith, we know that with complete markers and the usual CRRA/CES utility assumptions, we will get something like

$$
\begin{aligned}
\lambda^i p_t^i(s^t) &= c_t^i(s^t)^{-\sigma} \\
\lambda^j p_t^j(s^t) &= c_t^j(s^t)^{-\sigma}
\end{aligned}
$$

So the real exchange rate between i and j will be

$$
q_t^{ij}(s^t) \equiv \frac{p_t^j(z^t)}{p_t^i(z^t)} = \frac{\lambda^i c_t^j(s^t)^{-\sigma}}{\lambda^j c_t^i(s^t)^{-\sigma}} = \left(\frac{\lambda^i}{\lambda^j}\right)\left(\frac{c_t^i(s^t)}{c_t^j(s^t)}\right)^{\sigma}
$$

This implies that growth in relative consumption and growth in the real exchange rate change should be monotonically related with slope depending on σ. But this relationship fails **dramatically**: it is difficult to discern **any** relationship between changes in the real exchange rate and changes in relative consumption.

- So with complete markets and some non-tradability, we know that we get counter-factual predictions. One possibility, and one that Obstfeld and Rogoff endorse, is that the we can escape the Backus-Smith implications by dropping the complete markets assumptions. But the problem with this line of defense is that they claimed they could explain the six puzzles with goods market frictions only and without asset market frictions. Dropping the complete markets assumption is, however, exactly the same as imposing asset market frictions, so the rhetoric at least seems overblown.

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18 October 2004