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Home bias in equity portfolios

The issue at hand is the fact that in industrialized countries, consumers hold some 85-95% of their equity wealth in their own (i.e., home) assets. For example, in the late 1980s American consumers held some 94% of their equity wealth in the US stock market while Japanese consumers held some 98% of equity wealth in Japanese stocks. From the point of view of standard asset pricing theory, these do not seem like well diversified portfolios.

Although these home biases have falling over time (e.g., from about 94% to about 90% for Americans over the 1990s), the size of the bias remains big and difficult to explain.

A. Obstfeld and Rogoff's answer

Once again, Obstfeld and Rogoff's proposed solution hinges on **trade costs**. Consider a static two-country model where the representative consumer in the home country has expected utility preferences over $s \in \mathcal{S}$ possible states of nature.

$$\mathsf{E}\{U\} = \mathsf{E}\left\{\frac{C^{1-\rho}}{1-\rho}\right\} \equiv \sum_s \frac{C(s)^{1-\rho}}{1-\rho}\pi(s), \qquad \rho > 0$$

with CES consumption aggregate

$$C = \left[C_H^{\frac{\theta-1}{\theta}} + C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 0$$

The foreign country has symmetric preferences over a consumption aggregate C^*. The home country is endowed only with the home good, Y_H, while the foreign country is endowed only with the foreign good, Y_F.

Obstfeld and Rogoff allow costless asset market trade in a **complete set** of Arrow securities and imagine that there is an entirely symmetric (joint) distribution of endowments with typical realization $(Y_H(s), Y_F(s))$. In short, the preferences and endowments of the home and foreign country are **entirely** symmetric.

The sole friction in the model is the by-now familiar iceberg transportation costs that inhibit goods market trade. A proportion τ of goods shipped evaporate in transit. No-arbitrage in the goods market requires that the home and foreign prices of the home good satisfy

$$P_H(s) = (1-\tau)P_H^*(s)$$

(state by state). Similarly, the home and foreign prices of the foreign good must satisfy

$$(1-\tau)P_F(s) = P_F^*(s)$$

for each state $s \in \mathcal{S}$.

The relevant budget constraint for the home country is

$$\sum_s [P_H(s)C_H(s) + P_F(s)C_F(s)] \leq \sum_s P_H(s)Y_H(s)$$

and for the foreign country

$$\sum_s [P_H^*(s)C_H^*(s) + P_F^*(s)C_F^*(s)] \leq \sum_s P_F^*(s)Y_F(s)$$

If we let λ denote the Lagrange multiplier on the home country constraint and λ^* denote the multiplier on the foreign budget constraint, we have first order conditions

$$\begin{aligned}
\frac{\partial U(s)}{\partial C_H(s)}\pi(s) &= \lambda P_H(s), \qquad \frac{\partial U^*(s)}{\partial C_H^*(s)}\pi(s) = \lambda^* P_H^*(s) \\
\frac{\partial U(s)}{\partial C_F(s)}\pi(s) &= \lambda P_F(s), \qquad \frac{\partial U^*(s)}{\partial C_F^*(s)}\pi(s) = \lambda^* P_F^*(s)
\end{aligned}$$

for each state $s \in \mathcal{S}$. Because of the assumed symmetry of the problem, we will have equal Lagrange multipliers (i.e., equal welfare weights in the equivalent social planner's problem), so set $\lambda = \lambda^*$ and write the first order conditions as

$$\begin{aligned}
\frac{1}{P_H(s)}\frac{\partial U(s)}{\partial C_H(s)} &= \frac{1}{P_H^*(s)}\frac{\partial U^*(s)}{\partial C_H^*(s)} \\
\frac{1}{P_F(s)}\frac{\partial U(s)}{\partial C_F(s)} &= \frac{1}{P_F^*(s)}\frac{\partial U^*(s)}{\partial C_F^*(s)}
\end{aligned}$$

Using the goods market no-arbitrage conditions to eliminate the prices, we get

$$\begin{aligned}\frac{\partial U(s)}{\partial C_H(s)} &= (1-\tau)\frac{\partial U^*(s)}{\partial C_H^*(s)} \\ \frac{\partial U(s)}{\partial C_F(s)} &= \frac{1}{(1-\tau)}\frac{\partial U^*(s)}{\partial C_F^*(s)}\end{aligned}$$

If it were not for the trade costs (if $\tau = 0$) then we would have the familiar pure equalization of marginal utilities condition for a standard complete markets economy.

We can now compute the marginal utilities, for example

$$\begin{aligned}\frac{\partial U(s)}{\partial C_H(s)} &= C(s)^{-\rho}\left[C_H(s)^{\frac{\theta-1}{\theta}} + C_F(s)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}-1} C_H(s)^{\frac{\theta-1}{\theta}-1} \\ &= C(s)^{-\rho}\left[C_H(s)^{\frac{\theta-1}{\theta}} + C_F(s)^{\frac{\theta-1}{\theta}}\right]^{\frac{1}{\theta-1}} C_H(s)^{-\frac{1}{\theta}} \\ &= C(s)^{-\rho}C(s)^{\frac{1}{\theta}}C_H(s)^{-\frac{1}{\theta}}\end{aligned}$$

Doing this for all four marginal utilities allows us to write the optimality conditions as

$$C(s)^{\frac{1}{\theta}-\rho}C_H(s)^{-\frac{1}{\theta}} = (1-\tau)C^*(s)^{\frac{1}{\theta}-\rho}C_H^*(s)^{-\frac{1}{\theta}} \tag{1}$$

$$(1-\tau)C(s)^{\frac{1}{\theta}-\rho}C_F(s)^{-\frac{1}{\theta}} = C^*(s)^{\frac{1}{\theta}-\rho}C_F^*(s)^{-\frac{1}{\theta}} \tag{2}$$

The model is closed with the usual market clearing conditions, modified to account for the iceberg costs. These are, state by state,

$$C_H^*(s) = (1-\tau)[Y_H(s) - C_H(s)] \tag{3}$$

$$C_F(s) = (1-\tau)[Y_F(s) - C_F^*(s)] \tag{4}$$

For each state s, we have four equations in four unknowns, namely $\{C_H(s), C_F(s), C_H^*(s), C_F^*(s)\}$. These are not difficult to solve numerically, but to make much analytic progress we need to resort to a special case.

B. Separable special case

If we have the special case $\rho = 1/\theta$, the utility function is separable between $C_H(s)$ and $C_F(s)$ and the efficiency conditions (1)-(2) simplify to

$$\begin{aligned} C_H(s)^{-\frac{1}{\theta}} &= (1-\tau)C_H^*(s)^{-\frac{1}{\theta}} \\ (1-\tau)C_F(s)^{-\frac{1}{\theta}} &= C_F^*(s)^{-\frac{1}{\theta}} \end{aligned}$$

or

$$\begin{aligned} C_H^*(s) &= (1-\tau)^{\theta} C_H(s) \\ C_F(s) &= (1-\tau)^{\theta} C_F^*(s) \end{aligned}$$

Plugging these into the market clearing conditions and solving gives the **consumption allocations**

$$\begin{aligned} C_H(s) &= \frac{1}{1+(1-\tau)^{\theta-1}} Y_H(s) \\ C_F(s) &= \frac{(1-\tau)^{\theta}}{1+(1-\tau)^{\theta-1}} Y_F(s) \\ C_H^*(s) &= \frac{(1-\tau)^{\theta}}{1+(1-\tau)^{\theta-1}} Y_H(s) \\ C_F^*(s) &= \frac{1}{1+(1-\tau)^{\theta-1}} Y_F(s) \end{aligned}$$

As in any complete markets model, the individual consumption allocations (for each good) are fixed fractions of the world endowment of that good. Remember that each country is the sole supplier of its good, so the world endowment of the home good is just $Y_H(s)$ and the world endowment of the foreign good is just $Y_F(s)$. Notice that if we had zero trade costs, $\tau = 0$, then in this symmetric world each country would just have $C_H(s) = C_H^*(s) = \frac{1}{2}Y_H(s)$ and $C_F(s) = C_F^*(s) = \frac{1}{2}Y_F(s)$ state by state. Even with the trade cost, this model still has a fair amount of symmetry, however. For example, the home share of the foreign good is equal to the foreign share of the home good

$$\frac{C_F(s)}{Y_F(s)} = \frac{(1-\tau)^{\theta}}{1+(1-\tau)^{\theta-1}} = \frac{C_H^*(s)}{Y_H(s)}$$

and likewise the home share of the home good and the foreign share of the foreign good are the same

$$\frac{C_H(s)}{Y_H(s)} = \frac{1}{1+(1-\tau)^{\theta-1}} = \frac{C_F^*(s)}{Y_F(s)}$$

We are now ready to infer the **equity shares** for this complete markets economy. First note that because a proportion τ of goods evaporate in transit, the sum of the consumption allocations does not entirely exhaust the supplies of the goods. In the absence of trade costs, the implied equity and consumption shares would be the same (and in this symmetric world, equal to $\frac{1}{2}$). With trade costs, however, each country's share in the other country's output must be larger by a factor of $1/(1-\tau)$ than the corresponding consumption allocation if equity shares are to sum to one. That is, if we let $x_H(s)/Y_H(s)$ and $x_H^*(s)/Y_H(s)$ denote the home and foreign shares of the home output, we must have

$$\frac{x_H(s)}{Y_H(s)} = \frac{C_H(s)}{Y_H(s)} = \frac{1}{1+(1-\tau)^{\theta-1}} \qquad \text{all } s$$

but

$$\frac{x_H^*(s)}{Y_H(s)} = \frac{1}{1-\tau}\frac{C_H^*(s)}{Y_H(s)} = \frac{(1-\tau)^{\theta-1}}{1+(1-\tau)^{\theta-1}} \qquad \text{all } s$$

Of course, now we have

$$\frac{x_H(s)}{Y_H(s)} + \frac{x_H^*(s)}{Y_H(s)} = \frac{1}{1+(1-\tau)^{\theta-1}} + \frac{(1-\tau)^{\theta-1}}{1+(1-\tau)^{\theta-1}} = 1$$

Claims on the output of the home country exhaust the available supply.

Similarly, the home and foreign shares in foreign output are

$$\frac{x_F(s)}{Y_F(s)} = \frac{1}{1-\tau}\frac{C_F(s)}{Y_H(s)} = \frac{(1-\tau)^{\theta-1}}{1+(1-\tau)^{\theta-1}} \qquad \text{all } s$$

and

$$\frac{x_F^*(s)}{Y_F(s)} = \frac{C_F^*(s)}{Y_F(s)} = \frac{1}{1+(1-\tau)^{\theta-1}} \qquad \text{all } s$$

Recall that is there were no trade costs, $\tau = 0$, then all equity shares are equal to $\frac{1}{2}$, independent of the elasticity of substitution θ. But if we have Obstfeld and Rogoff's benchmark case of $\tau = 0.25$ and $\theta = 6$ then

$$\begin{aligned}
\frac{x_H(s)}{Y_H(s)} &= \frac{1}{1+(0.75)^5} = 0.8082 \\
\frac{x_H^*(s)}{Y_H(s)} &= \frac{(0.75)^5}{1+(0.75)^5} = 0.1918
\end{aligned}$$

Clearly, even in this very symmetric world, we can get considerable home bias in equity shares.

If we have lower trade costs, say $\tau = 0.10$, but are prepared to go with $\theta = 10$ (and you may recall that some industries have estimated θ in the vicinity of 20, so this is not necessarily such a large elasticity of substitution), then we get equity shares

$$\begin{aligned}
\frac{x_H(s)}{Y_H(s)} &= \frac{1}{1+(0.90)^9} = 0.7208 \\
\frac{x_H^*(s)}{Y_H(s)} &= \frac{(0.90)^9}{1+(0.90)^9} = 0.2792
\end{aligned}$$

This is still a considerable bias.

In short, even with a frictionless asset market, we may be able to explain a high home bias in equity shares by appealing to moderate trade costs and a moderate elasticity of substitution between goods. As in their other examples, Obstfeld and Rogoff have used a very symmetric example which forces all of the "heavy lifting" to be done by the trade costs. If they had non-separable preferences ($\rho \neq \frac{1}{\theta}$) or home bias in preferences or asymmetric endowment distributions, etc, then they would not even have to appeal to trade costs as large as $\tau = 0.25$ to get considerable home bias.

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