316-406 ADVANCED MACROECONOMIC TECHNIQUES **Final samples**

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The final will last 180 minutes and has two questions. The first question is worth 120 marks, while the second question is worth only 60 marks. Within each question there are a number of parts and the weight given to each part will also be indicated.

Here are two sample questions.

Question 1. *Term premia* (120 marks). Mehra and Presoctt study the equity premium, the average excess return on equity over bonds. In this question, you will study the term premium, the difference between the returns on bonds of differing maturity. Consider a representative agent consumption based asset pricing model where preferences are

$$\mathsf{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\gamma}}{1-\gamma}\right\}, \qquad 0<\beta<1 \text{ and } \gamma\geq 0$$

There are two kinds of assets. First, there is a "Lucas tree" with dividends $\{y_t\}$ with gross growth rate that follows a Markov chain. That is, let

$$x_{t+1}=\frac{y_{t+1}}{y_t}$$

Then $\{x_t\}$ follows a Markov chain with transition probabilities

$$\pi(x',x)=\Pr(x_{t+1}=x'|x_t=x)$$

Suppose the representative agent can trade in shares in the tree (with constant exogenous supply normalized to 1) and can trade in one and two period bonds. A 1-period bond is a riskless claim to a unit of consumption to be delivered next period, while a 2-period bond is a riskless claim to one unit of consumption to be delivered in two period's time.

(a) (20 marks): Let $q_j(x,y)$ denote the price of a j-bond ($j=1,2$) if the current aggregate state is (x,y) and let $p(x,y)$ denote the price of a claim to the Lucas tree. Let $V(w,x,y)$ denote the consumer's value function if their individual wealth is w and the aggregate state is (x,y). Write down a Bellman equation for the consumer's problem. Be careful to explain the Bellman equation and any constraints that you provide. [*Hint*: a 2-period bond bought this period can be re-sold as a 1-period bond next period].

(b) (15 marks): Define a **recursive competitive equilibrium** for this economy.

(c) (15 marks): Go as far as you can in solving for the price of the tree, $p(x, y)$. Carefully explain how you could implement this solution on a computer. In your answer, let the Markov chain have $i = 1, ..., n$ states.

(d) (15 marks): Use first order and envelope conditions to characterize the optimal decisions of the representative consumer. Using these and market clearing conditions, solve for the prices $q_j(x, y)$ for $j = 1, 2$. Give economic intuition for your solutions. Explain the difference (if any) between the price of a 2-period bond and the price of two 1-period bonds. [*Hint*: what would these prices be if the consumer was risk neutral ($\gamma = 0$)? How (if at all) does this change when the consumer is risk averse ($\gamma > 0$)?]

(e) (30 marks): Define bond **returns** by the formula $R_j(x, y) \equiv [1/q_j(x, y)]^{1/j}$. Provide solutions for bond returns. For given state (x, y), explain whether $R_1(x, y) \geq R_2(x, y)$ or not. Give as much economic intuition as possible. Again, it might be useful to consider the risk neutral case as a benchmark and then explain how (if at all) your answer changes when the consumer is risk averse.

(f) (25 marks): The **forward price** f of a 2-period bond (i.e., the price of a 2-period bond that can be locked in safely one period in advance) is given by

$$f(x, y) \equiv \frac{q_2(x, y)}{q_1(x, y)}$$

The **holding period return** h on a 2-period bond that is bought at $q_2(x, y)$ and held for one period and then sold at $q_1(x', y')$ is

$$h(x', y', x, y) \equiv \frac{q_1(x', y')}{q_2(x, y)}$$

Using your answers from part (d), provide solutions for the forward price and holding period return. Go as far as you can in explaining the stochastic pattern you would expect to see in forward prices and holding period returns. Again, explain how your answer depends on the degree of risk aversion.

Question 2. *Solving the stochastic growth model* (60 marks). Consider a planner with the problem of maximizing

$$\mathsf{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\}, \qquad 0 < \beta < 1$$

subject to a resource constraint

$$c_t + k_{t+1} = z_t f(k_t) + (1 - \delta) k_t, \qquad 0 < \delta < 1$$

and the non-negativity constraints

$$c_t \geq 0, \quad k_t \geq 0$$

where c_t denotes consumption, k_{t+1} denotes capital carried into the next period, δ denotes a constant depreciation rate, and z_t is the level of technology, which follows a Markov chain on a discrete set $\mathcal{Z}$ with transitions given by

$$\pi(z', z) = \Pr(z_{t+1} = z' | z_t = z)$$

(a) (10 marks): Let $V(k, z)$ denote the value function. Set up a Bellman equation for this dynamic programming problem.

(b) (10 marks): Use first order and envelope conditions to characterize the solution to this problem.

(c) (10 marks): Give an algorithm that explains how you would find approximate solutions by value function iteration on a discrete state space. In your answer, let $\mathcal{K} \times \mathcal{Z}$ denote the discretized state space.

(d) (20 marks): Suppose that $k' = g(k, z)$ denotes the policy function that you obtain from solving your dynamic programming problem. Let $\mu_t(k, z)$ denote the unconditional distribution of (k, z) pairs on $\mathcal{K} \times \mathcal{Z}$. That is,

$$\mu_t(k, z) = \Pr(k_t = k, z_t = z)$$

Explain how you can use the policy function $g(k, z)$ and the transitions $\pi(z', z)$ to create a law of motion that maps $\mu_t(k, z)$ to $\mu_{t+1}(k', z')$. Give an algorithm that explains how you could solve for a stationary distribution [i.e., a time-invariant $\mu(k, z)$].

(e) (10 marks): Suppose that $\mathcal{Z} = \{z_L, z_H\}$ with $z_L < z_H$. If the utility and production functions have the usual properties (strictly increasing, strictly concave, etc) sketch the policy functions $k' = g(k, z_L)$ and $k' = g(k, z_H)$ on a "45-degree" phase diagram. Explain how you could determine which subset $[\underline{k}, \overline{k}]$ of $\mathcal{K}$ has positive probability in the stationary distribution.

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21 October 2004