316-406 Advanced Macroeconomic Techniques

Problem Set #5 *Due 12 November, 2004*

Question 1. (*Equity Premium Puzzle*). Consider the Mehra-Prescott model as described in the notes. The (gross) growth rate of dividends $x' \equiv y'/y$ follows a symmetric 2-state Markov chain with states

$$\begin{aligned} x_1 &= 1 + 0.018 - 0.036 \\ x_2 &= 1 + 0.018 + 0.036 \end{aligned}$$

and symmetric transition probabilities

$$\begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} = \begin{pmatrix} 0.43 & 0.57 \\ 0.57 & 0.43 \end{pmatrix}$$

where

$$\pi_{ij} = \Pr(x_{t+1} = x_j | x_t = x_i), \qquad i, j = 1, 2$$

The Bellman equation for the household is

$$V(w, x, y) = \max_{s', B'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + 0.96 \sum_{x'} V(w', x', y') \pi(x', x) \right\}$$

where w denotes household wealth and the maximization on the right hand side is subject to a budget constraint

$$c + p(x, y)s' + q(x, y)B' \leq w$$

and the laws of motion

$$\begin{aligned} w' &= [p(x', y') + y']s' + B' \\ y' &= x'y \end{aligned}$$

- Let $\sigma = 1.5$. Solve for equilibrium bond prices $q(x, y)$ and equity prices $p(x, y)$. Starting from an initial distribution of $\pi_0 = \bar{\pi} = (0.5, 0.5)$, iterate on the Markov chain for dividend growth for $t = 1, ..., 100$. Calculate the realized bond prices, bond returns and equity prices and equity returns. Plot each of these variables and describe the equilibrium stochastic processes you observe.

- Compute the equity premium as defined by Mehra and Prescott.

- Repeat the previous exercises for $\sigma = 5, 10, 20$. Explain how your answers differ as you increase relative risk aversion. Give economic intuition for your findings.

Question 2. (*Incomplete Markets*). Consider an Aiyagari model where the typical household has Bellman equation

$$V(k,n)=\max_{k'\geq 0}\left\{\frac{c^{1-\gamma}}{1-\gamma}+\beta\mathsf{E}\{V(k',n')|n\}\right\},\qquad 0<\beta<1$$

where the maximization on the right hand side is subject to the budget constraint

$$c+k'\leq(1+r-\delta)k+wn,\qquad 0<\delta<1$$

Log labor supply follows an exogenous AR(1) process

$$\log(n')=\rho\log(n)+\sigma(1-\rho^2)^{1/2}\varepsilon',\qquad 0<\rho<1 \text{ and } \sigma>0$$

where the innovations ε' are standard normal random variables.

The typical firm has production function

$$F(K,N)=K^{\alpha}N^{1-\alpha},\qquad 0<\alpha<1$$

The parameters values are

Symbol	Meaning	Value
β	time discount factor	0.96
α	capital's share in national output	0.36
δ	depreciation rate of physical capital	0.08
γ	coefficient of relative risk aversion	3.00
σ	standard deviation of log labor supply	0.40
ρ	autocorrelation of log labor supply	0.60

- Define a stationary recursive competitive equilibrium for this economy.
- Calibrate a 2-state Markov chain representation for $\log(n)$. That is, choose parameters of the 2-state Markov chain so that the mean, variance, and autocorrelation of the Markov chain equal their counterparts for the AR(1) process.
- Solve for a stationary recursive competitive equilibrium when log labor supply follows the calibrated 2-state Markov chain.
- Compute an approximation to the policy function $k'=g(k,n)$. Plot $g(k,n_H)$ and $g(k,n_L)$ where $n_L<n_H$ are the two calibrated states of the Markov chain.
- Compute the equilibrium invariant distribution and plot each of $\mu(k,n_H)$ and $\mu(k,n_L)$. Describe the results you find.
- Compute the equilibrium interest rate (i.e., $r-\delta$) and savings rate (i.e., $\delta K/F(K,N)$) for this incomplete markets economy. How do they compare to the corresponding values for the complete markets benchmark? [*Hint*: what is the interest rate in the steady state of the neoclassical growth model].