316-406 Advanced Macroeconomic Techniques

Problem Set #3 *Due 4 October, 2004*

For this problem set you should use Harald Uhlig's Matlab "toolkit" for solving log-linear models. Save Uhlig's files "`solve.m`" and "`options.m`" to your local directory, then follow the example in my program "`stochastic_growth.m`" to set up the coefficients. All of these files will be available on the class website.

Question 1. (*Real Business Cycles*). Consider the social planning problem of maximizing utility

$$\mathsf{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\log(c_t) + \log(\ell_t)] \right\}$$

subject to a resource constraint

$$c_t + k_{t+1} = z_t k_t^{\alpha} n_t^{1-\alpha} + (1-\delta) k_t, \qquad k_0 \text{ given}$$

and a constraint on the time endowment

$$n_t + \ell_t = 1$$

Let log technology follow an AR(1),

$$\log(z_{t+1}) = \rho \log(z_t) + \varepsilon_{t+1}, \qquad 0 < \rho < 1$$

where $\{\varepsilon_{t+1}\}$ is Gaussian white noise with initial realization z_0 given.

- Derive first order conditions that characterize optimal choices of consumption, employment, and capital accumulation.
- Let the parameters of the model be

Symbol	Meaning	Value
β	time discount factor	0.99
α	capital's share in national output	0.33
δ	depreciation rate of physical capital	0.04
ρ	serial correlation of technology shock	0.95

  Solve for the non-stochastic steady state.

- Log-linearize the model around the non-stochastic steady state. Show that the log-linear model can be written in the form

$$\begin{aligned} 0 &= AX_t + BX_{t-1} + CY_t + DZ_t \\ 0 &= \mathsf{E}_t\{FX_{t+1} + GX_t + HX_{t-1} + JY_{t+1} + KY_t + LZ_{t+1} + MZ_t\} \\ Z_{t+1} &= NZ_t + \varepsilon_{t+1} \end{aligned}$$

  Provide explicit solutions for each of the coefficients, $A, B, C, ..., N$. In these equations, X_t contains the endogenous state variables, Y_t contains the control variables, and Z_t contains the exogenous state variables. As part of your answer, you will need to explain exactly which variables from the model are in each of X_t, Y_t and Z_t.

Question 2. (*Uhlig's Toolkit*). Guess that a solution takes the form

$$\begin{aligned} X_t &= PX_{t-1} + QZ_t \\ Y_t &= RX_{t-1} + SZ_t \end{aligned}$$

for unknown coefficient matrices P, Q, R, S. Use Harald Uhlig's Matlab "toolkit" to solve for these coefficients matrices.

Question 3. (*Impulse Responses*). Use your answers to compute the effect of a one-time shock to the level of productivity. That is, set the value of $\varepsilon_0 = 1$ and $\varepsilon_t = 0$ for $t \geq 1$ and trace out the effects on productivity, consumption, employment, investment and output. Graph your answers for $t = 0, 1, ..., 50$. Briefly explain your answers.

Question 4. (*Simulations*). For $t = 1, ..., 1000$, sample random draws for $\{\varepsilon_{t+1}\}$ and iterate on the laws of motion to compute paths for $\{X_t\}$, $\{Y_t\}$ and $\{Z_t\}$. Then drop the first 500 observations of each series and compute the standard deviations of each of the variables over the remaining $t = 501, ...1000$ observations. Explain why you drop these initial values. Report the standard deviation of each variable as a ratio to the standard deviation of output. Explain your answers.