

Please don't go "over the top" in answering the following questions. In most cases, a couple of numbers, or a few words or a picture is all that is required. You are encouraged to use a program like Matlab or Gauss for this exercise, though in actual fact a simple spreadsheet like Excel is all that is really required.

Question 1. A discrete-time version of the Ramsey growth model leads to a pair of non-linear difference equations in consumption c_t and the capital stock (see Note 2a for more detail)

$$c_t + k_{t+1} = f(k_t) + (1 - \delta)k_t \quad (1)$$

$$U'(c_t) = \beta U'(c_{t+1})[1 + f'(k_{t+1}) - \delta] \quad (2)$$

with two boundary conditions, the given initial condition k_0 and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t U'(c_t) k_{t+1} = 0$$

(and non-negativity constraints $c_t \geq 0$ and $k_t \geq 0$). Now let the period utility function be

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma > 0$$

and let the production function be

$$f(k) = zk^\alpha, \quad z > 0, \text{ and } 0 < \alpha < 1$$

Show that the resource constraint (1) and the Euler equation (2) of the log-linear version of this model can be written, respectively,

$$0 = A\hat{k}_{t+1} + B\hat{k}_t + C\hat{c}_t$$

$$0 = F\hat{k}_{t+1} + G\hat{k}_t + J\hat{c}_{t+1} + K\hat{c}_t$$

where the coefficients A, B, C, \dots are functions of the parameters of the model. Provide explicit solutions for these coefficients.

Question 2. Guess that a solution to the log-linear model is of the form

$$\hat{k}_{t+1} = P\hat{k}_t$$

$$\hat{c}_t = R\hat{k}_t$$

for unknown coefficients P and R . Use the "method of undetermined coefficients" to solve for the unknown coefficients P and R in terms of the known coefficients A, B, C, \dots etc. Now let the parameters have the values

Symbol	Meaning	Value
α	capital's share in national output	0.33
δ	annual depreciation rate of physical capital	0.04
β	time discount factor	0.97
σ	curvature of utility function	2.00
z	level of technology	1.00

Use these numbers to first solve for the steady state values \bar{c} and \bar{k} , then solve for the coefficients A, B, C, \dots etc and finally solve for the unknown coefficients P and R .

Question 3. (*Recovery from a war*). Let the initial capital stock be 10% below steady state. What value does \hat{k}_0 have? Using this value, plot the time paths of consumption \hat{c}_t and capital \hat{k}_{t+1} for $t = 0, 1, 2, \dots, T$ where T is large enough to ensure that \hat{k}_T is close to zero. Also plot the implied time paths of investment and output. Explain why the time paths look the way they do. Give **economic** intuition wherever you can. How long does it take until $\hat{k}_t = 0.5\hat{k}_0$?

Question 4. (*Technological breakthrough*). Suppose at $t = -1$, the economy is in steady state as in Question 2. Now suppose that at $t = 0$, a technological revolution takes place and z increases from $z = 1$ to $z = 2$. This **shifts the steady state** to new levels \bar{c}' , \bar{k}' . Compute these new steady state values. What is the implied deviation of \hat{k}_0 relative to this new steady state? As in Question 3, trace out the time paths of \hat{c}_t and capital \hat{k}_{t+1} as they adjust to this new steady state. Also plot the implied time paths of investment and output. Explain why the time paths look the way they do. Again, give economic intuition wherever you can.

Question 5. In the decentralized version of this model, the wage rate and rental rate of capital are given by

$$\begin{aligned}w_t &= f(k_t) - f'(k_t)k_t \\r_t &= f'(k_t)\end{aligned}$$

Using the assumed production function, $f(k) = zk^\alpha$, solve for these factor prices. Then express the factor prices as log-linear equations in terms of \hat{k}_t . Use the results from Questions 3 and 4 to solve for the implied path of wage rates \hat{w}_t and rental rates on capital \hat{r}_t in each case. Again, explain why these time paths look the way they do.

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